



The program that takes the struggle out of math

Level 3 Teacher's Manual Sample

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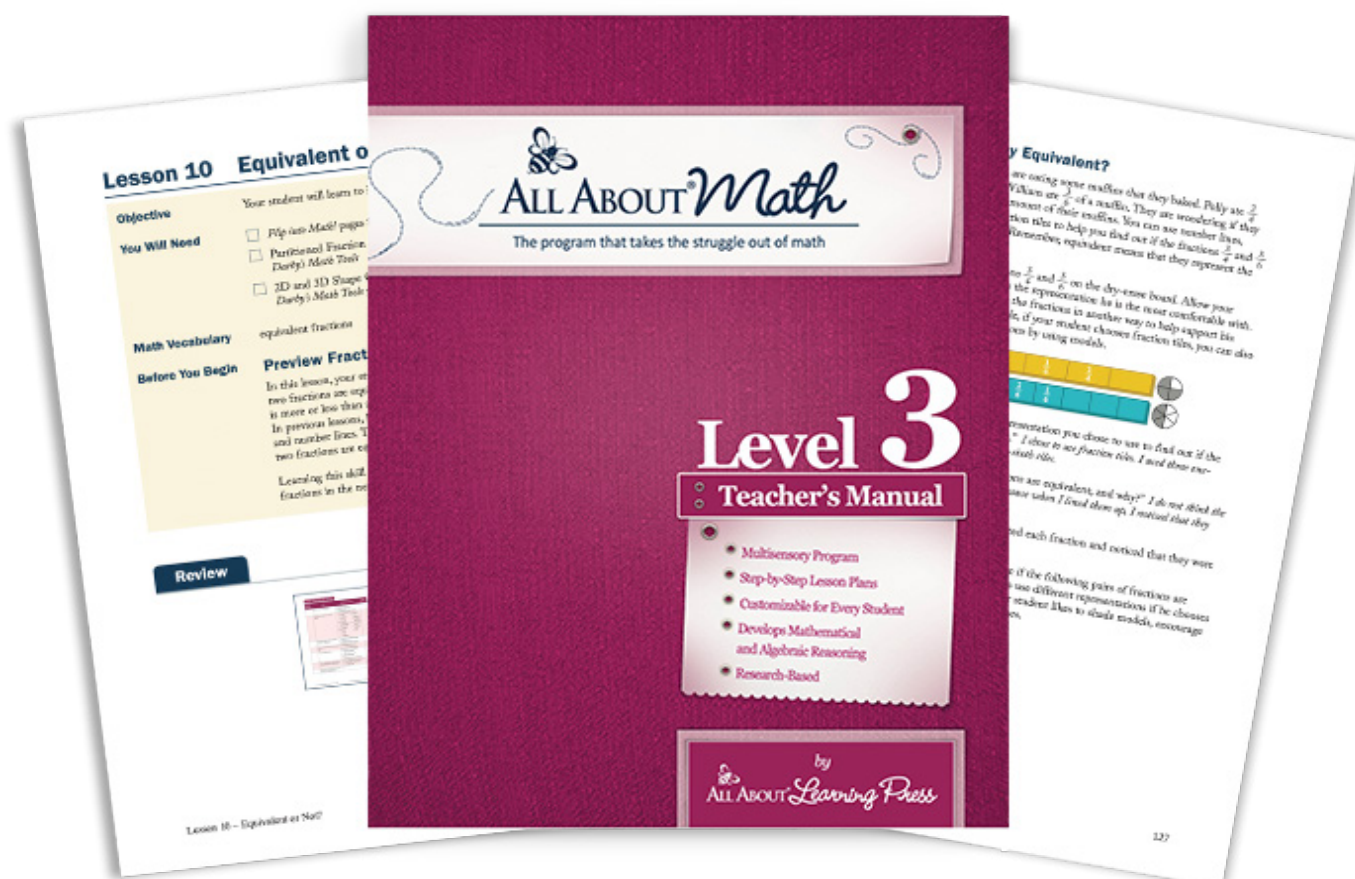


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1

Preparing for Level 3

Start Here!

To prepare for teaching *All About Math* Level 3, you can either watch our short videos or follow the checklist on the subsequent pages. Do whichever works best for you!

Option 1: Watch the Videos



Go to www.aalp.tv/math-level-3 on your phone, tablet, computer, or scan the QR code to be taken directly to the videos.



Let us show you how to get set up for success!



After watching the videos, turn to page 33 of this teacher's manual to start teaching the first lesson.



Option 2: Read the Following Pages



Check off each as you complete it.



Is Your Student in the Right Level?

If your student did not complete *All About Math* Level 2, use this checklist and the *Flip into Math!* activity book pages 387 to 393 to verify placement in Level 3. Your student should get all items in a question correct in order to checkmark that question. Your student will need a ruler that shows both centimeters and inches.

- ☐ 1. Your student can count in the following ways:

- **count within 1,000, starting with the number 793 and ending with 850**
- **count backward from 20 to 0**
- **skip count by 5's, 10's, and 100's, for example, 10, 20, 30, etc.**

- ☐ 2. Your student can solve addition and subtraction story problems and provide an equation to match each. To test this, read the story problems in question 2 aloud, one at a time. Your student may use objects or drawings to help solve the story problems and write matching equations.

stickers: $8, 9 + 8 = 17$ or $17 - 9 = 8$

free throws: $5, 12 - 7 = 5$ or $7 + 5 = 12$

- ☐ 3. Your student can find the sum of three addends within a story problem by breaking apart numbers to make a ten. To test this, read the story problem in question 3 aloud. Your student may use objects or drawings to help solve the story problem and explain how she found the answer.

Pieces of fruit: *14, I broke down the 6 bananas into $5 + 1$ so that I could make a 10 by adding the 5 bananas with the 5 apples. Then, there is 1 more banana and 3 oranges, which makes 4 pieces of fruit. So, 10 plus 4 equals 14.*

- ☐ 4. Your student can add multi-digit numbers, including composing a ten. Have your student solve each problem on the student activity page.

$65 + 29 = \underline{\quad}$ (Answer: 94)

$457 + 283 = \underline{\quad}$ (Answer: 740)

- ☐ 5. Your student can subtract multi-digit numbers, including decomposing a ten. Have your student solve each problem on the student activity page.

$96 - 48 = \underline{\quad}$ (Answer: 48)

$703 - 254 = \underline{\quad}$ (Answer: 449)

- ☐ 6. Your student can read, write, and represent multi-digit numbers in different ways. Have your student fill in the blanks on the student activity page. Then, have your student read aloud each number.

Row 1: 586, “five hundred eighty-six”

Row 2: $300,000 + 50,000 + 1,000 + 700 + 40 + 2$, “three hundred fifty-one thousand, seven hundred forty-two”

- ☐ 7. Your student understands place value and can use it to tell you the value of each digit in a number. For example, the value of the 4 in the number 45,678 is 40,000 and the value of the 7 is 70. To test this follow the scripting example shown below.

“What is the value of the 2 in the number 253?” (*Answer: 200*)

“What is the value of the 5 in the number 253?” (*Answer: 50*)

“What is the value of the 3 in the number 253?” (*Answer: 3*)

Continue to test your student using the following number:

745,302 (*Answers: 700,000, 40,000, 5,000, 300, 2*)

- ☐ 8. Your student can compare two multi-digit numbers by using comparison symbols ($>$, $<$, $=$) to show greater than, less than, or equal to. To test this, have your student compare each pair of numbers on the student activity page.

Answers: $791 > 591$, $2,048 < 2,100$, $5,987 = 5,987$

- ☐ 9. Your student can order a set of three multi-digit numbers. Have your student list the numbers in the correct order on the student activity page.

3,902 3,745 5,124 (*Answer: 3,745; 3,902; 5,124*)

7,455 7,031 7,458 (*Answer: 7,458; 7,455; 7,031*)

- ☐ 10. Your student can mentally add and subtract tens and hundreds to a given number. Have your student solve each expression on the student activity page.

$134 + 50$ (*Answer: 184*) **$652 - 30$** (*Answer: 622*) **$845 - 600$** (*Answer: 245*)

- ☐ 11. Your student can estimate and measure length using a ruler. To test this, have your student estimate and measure the length in centimeters of the images on the student activity page.

Feather (*possible estimate range: 3 - 7 cm, actual measurement: 5 cm*)

- ☐ 12. Your student can compare standard units of length, including inches and feet. To test this, ask your student which unit would be best to measure the length of the following items in your home. Then have your student use the ruler to find the actual length in inches or feet.

pencil (*Answer: inches, actual length will vary*),

table (*Answer: feet, actual length will vary*)

- ☐ 13. Your student can solve one and two-step story problems involving length and provide an equation to match each. To test this, read the story problems in question 13 aloud, one at a time. Your student can use objects or drawings to help solve the story problems and write matching equations.

yarn: $65 - \underline{\hspace{1cm}} = 47$; 18 cm

walk to kitchen: $31 + \underline{\hspace{1cm}} + 47 = 92$; 14 ft

- ☐ 14. Your student can identify the time shown on an analog clock using a.m. and p.m. On the student activity page, have your student write the time shown on each clock. Then, have her circle the correct phrase to complete the statement and tell the exact time.

Clock 1: (Answer: *quarter past 5 o'clock, 5:15*)

Clock 2: (Answer: *half past 9 o'clock, 9:30*)

Clock 3: (Answer: *quarter 'til 2 o'clock, 1:45*)

Then, have your student identify if the time is a.m. or p.m. To test this, follow the scripting example shown below.

“At 5:15, we might be playing outside. Is it a.m. or p.m.?” (Answer: *p.m.*)

“At half past 9, we might be finishing up breakfast. Is it a.m. or p.m.?” (Answer: *a.m.*)

“At 1:45 we are all sound asleep. Is it a.m. or p.m.?” (Answer: *a.m.*)

- ☐ 15. Your student can identify the name and value of each coin and determine the total value of a group of coins. To test this, first follow the scripting example shown below.

“Point to the dime. What is the value of one dime?” (Answer: *10 cents*)

“Point to the quarter. What is the value of one quarter?” (Answer: *25 cents*)

“Point to the penny. What is the value of one penny?” (Answer: *1 cent*)

“Point to the nickel. What is the value of one nickel?” (Answer: *5 cents*)

Then have your student identify the total value represented by the group of coins on the student activity page.

Answer: 86 cents

- ☐ 16. Your student can solve addition and subtraction story problems about money. To test this, read the story problem in question 16 aloud. Your student can use objects or a drawing to help solve the story problem.

three fruits: No, he needs 16 cents.

- ☐ 17. Your student can round numbers to the nearest 10 or 100. Read aloud each statement and have your student round to fill in the blanks on the student activity page. Your student can use a number line to solve these problems.
- “578 rounded to the nearest 10 is _____, but rounded to the nearest 100 is _____.” (*Answer: 580, 600*)
 - “Rounded to the nearest 10, the numbers 56, 59, and 63 all round to _____.” (*Answer: 60*)

How did your student do?

- If your child could easily complete 15 or more of the 17 skills, begin Level 3.
- If just one or two areas were difficult, you can remediate in those areas as you start Level 3.
- If 14 or fewer boxes were checked, start with Level 2 to build a strong foundation for math.

If you have any questions about the program or would like to learn how to adapt certain aspects of the program to accommodate your child’s needs, feel free to call us at 715-477-1976 or email us at support@allaboutlearningpress.com. And if you need ideas on how to help your child build skills, just let us know—we are always happy to help!



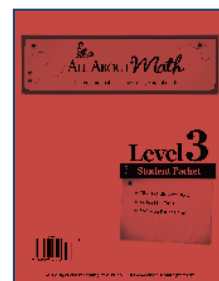
Gather the Materials

In addition to this teacher's manual, you will need the following items:

1 Student Packet

The Student Packet contains:

- *Flip into Math!* activity book
- Stickers for the Progress Chart
- *Darby's Math Tools* (See page 27, Preview *Darby's Math Tools*, for more details)



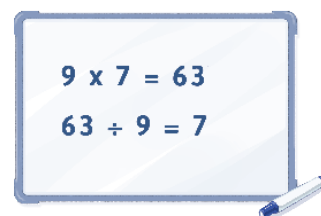
2 All About Math Manipulatives Kit

The manipulatives kit includes hands-on materials to support learning. See page 25, Learn about Manipulatives, for more details about the manipulatives for Level 3.



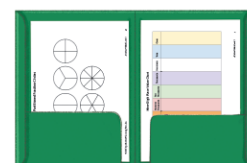
3 Dry-erase board and Markers

You can use any size. We recommend a hand-held dry-erase board for ease in demonstrating concepts. However, if you already have a dry-erase board for *All About Reading* or *All About Spelling*, you can also use your existing board.



4 Folder (Optional)

It's helpful to have a folder for storing *Darby's Math Tools*. You may also want a folder for storing *Darby's Math Fun!* games so they can be replayed.





The *All About Math* Method

First of all, you can do this! *All About Math* is a scripted, open-and-go program developed for busy parents, teachers, and tutors who want to teach mathematics in the most effective way possible. This program doesn't require long periods of study, you don't have to develop your own lesson plans, and you don't have to stress over what to teach next—because everything is laid out for you, step-by-step. You'll get a solid grounding in how to teach mathematics without being overwhelmed.

Your student will be actively involved in the learning process. This is a truly multisensory program; your student will learn through sight, sound, and touch. Everything is taught in context, and your student will apply what he has learned right away. Your student will be engaged in thinking, processing, comparing, and learning.

Students who use the *All About Math* method tend to feel a sense of excitement in learning. And they should! They are learning how to think, explore, and grow in their abilities. They will feel successful as they see continual progress.

There are no gaps in this program. *All About Math* teaches your student everything he needs to know to build a strong foundation of numeracy, operation, and algebraic thinking. Each concept builds upon the previous one, ensuring a comprehensive understanding that leverages existing knowledge.

***All About Math* acknowledges the diverse needs of learners and addresses the five key components of effective instruction:**

1. **Strong Conceptual Understanding:** We connect mathematical concepts, fostering a deeper understanding that transcends memorization.
2. **Procedural Fluency and Skills:** Students master essential skills like multiplication, division, and fractions through practice and application.
3. **Communication and Collaboration:** We encourage students to explain their reasoning, fostering collaboration and clear communication through discussions and activities.
4. **Assessment and Differentiation:** Our program offers regular assessments so you can see how your student is doing. It allows you to cater to individual needs by offering differentiated instruction; instruction that allows you to adjust the pace, complexity, and activities to your student's needs.
5. **Positive Learning Environment:** We encourage students to believe in their ability to learn and grow through perseverance and effort.

***All About Math* is a mastery-based program.** As such, the levels don't necessarily correspond to grade levels. In mastery-based learning, students master foundational concepts before moving on to more advanced concepts, regardless of age or grade level. Some concepts will take many lessons to master. The instructions in each lesson help you know whether to move on, while the concept reminders on the *Daily Review Tracker* help you continue to work toward mastery.

Most importantly, *All About Math* is committed to results. The *All About Math* program has a very focused mission: to enable you to teach your student mathematics while guaranteeing retention and enjoyment. Our approach to mathematics focuses on enabling students to become confident, fluent mathematicians who can absorb and retain new information.

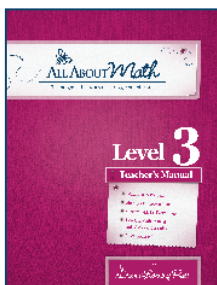
If you ever have a question as you are teaching, please feel free to contact us at support@allaboutlearningpress.com or 715-477-1976.

We're here to help!



Preview the Teacher's Manual

As you flip through the teacher's manual, you'll notice that all the lessons are laid out for you step-by-step. You'll also find two types of lessons:



• **New Concept Lessons:** In these lessons, your student will learn new skills and concepts. You can see an example of a typical “New Concept” lesson in Lesson 3 on page 53.

• **Progress Monitoring Lessons:** In the Show What You Know! lessons, your student will review and practice the new concepts taught in the previous lessons. You can see an example of a typical Progress Monitoring lesson in Lesson 14 on page 161.

Each new concept lesson consists of six parts:

1. **Before You Begin:** This cream-colored box contains an overview of the lesson and is meant only for you, the teacher. Reading it takes only a few minutes, after which you'll be well-equipped to teach the lesson confidently.
2. **Review:** You will begin the lesson by reviewing concepts learned previously, giving your student a quick review of skills or concepts essential to the new learning. Starting in lesson 5, you will need your student's *Daily Review Tracker* for this part of the lesson.
3. **New Teaching:** This is the hands-on, multisensory portion of the lesson. Your student will work with the manipulatives as you gradually introduce new concepts. Scaffolding techniques such as modeling, guided practice, and feedback help students progress at their own pace and achieve deeper understanding.

Then, your student will use activity sheets as she continues to practice the new concepts. The activities encourage teachers to highlight connections, helping students see the bigger picture and develop a more coherent understanding of mathematical concepts.

Finally, Math Reflections encourage your student to ask questions and express her understanding. This allows the teacher to identify any misconceptions and address them directly.

(See page 19, Math Reflections and Dialogue, for more details)

4. **Extended Practice:** Optional activities are included for students who need more practice. By revisiting and practicing the skills in different ways if needed, students develop fluency and automaticity, allowing them to solve problems and perform calculations with greater accuracy and speed.
5. **Darby's Math Fun!:** Fun and engaging activities provide opportunities for students to use and apply the new concepts they have learned in a meaningful context. This helps them move the information from short-term to long-term memory, strengthening their understanding

and improving their ability to recall and apply concepts later. These activities are designed to encourage playing more than once to reinforce concepts and skills.

6. **Track Your Progress:** At the end of each lesson, record your student's progress on the Progress Chart.

Take a few minutes to flip through the Appendices section starting on page 467. The Appendices include a few extra resources to help you and your student get the most out of your math lessons.



Math Reflections and Dialogue

It's incredibly important for children to talk about what they are learning in math. Verbalizing their thinking helps deepen their understanding, build critical reasoning skills, and strengthen their ability to communicate complex ideas. That is why you will find “Math Reflection” sections in every *All About Math* Lesson, and you will also see dialogue encouraged throughout. Here are some key benefits of encouraging math discussions:

Math Reflection

“Let’s Reflect!”

Ask some questions to guide your student’s reflection:

- “What does it mean when two fractions have the same numerator?”
- “What are some different ways you can represent fractions?”
- “What is something that challenged you today?”

This section is located after the Complete Activity Sheet section in each New Concept Lesson and after the last question in each Progress Monitoring Lesson.

Deepens Understanding: When your student talks through a problem, he is forced to clarify his thinking. Explaining his reasoning helps solidify the concepts in his own mind, making it easier for him to understand and retain the material. Talking through math problems can also reveal misunderstandings or gaps in knowledge. If he is unable to explain his thinking, it may highlight areas where he will need further instruction or support.

Encourages Active Engagement: Math discussions help your student move from passively receiving information to actively engaging with the material. When he verbalizes his thought processes, he is more likely to take ownership of his learning and develop a deeper connection to the content.

Promotes Critical Thinking and Problem Solving: Talking about math encourages your student to reason logically and justify his thinking. Discussing different strategies and approaches fosters critical thinking and can lead to deeper insights and a broader range of strategies for solving problems.

Enhances Mathematical Vocabulary: Talking about math helps your student develop and expand his mathematical vocabulary and encourages him to use specific, accurate language, which reinforces his understanding of the terms and concepts involved.

Improves Memory and Retention: When your student talks about math, he is engaging both the verbal and cognitive centers of the brain, which enhances memory and understanding. Explaining concepts to others forces him to organize and articulate his knowledge in a coherent way, and reinforces learning and retention.

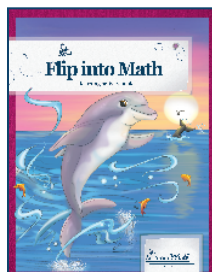
Encourages a Positive Attitude Toward Math: By encouraging your student to talk about his learning, we help him see math as a subject he can engage with and discuss, rather than a subject that is difficult or intimidating. Positive discussions about math help develop a healthy attitude toward the subject and can reduce math anxiety.

Talking about math in real-world contexts or through stories can help him see the relevance and practical applications of what he is learning, making math more engaging and meaningful.



Preview the Activity Book

The *Flip into Math!* activity Book contains:

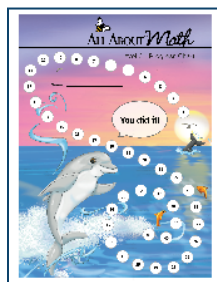


- Progress Chart
- *Daily Review Tracker*
- Activity Sheets
- Darby's Math Fun!
- Math Practice
- Story Problems and Situations
- Certificate of Achievement

The lesson plans in the teacher's manual will tell you which pages you need for each lesson. The pages in the activity book are perforated for easy removal.

Let's take a quick look at each part of the activity book.

Progress Chart



The *Progress Chart* can be found on page 5 of the activity book.

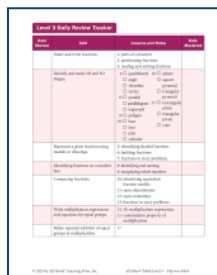
This chart is a fun and encouraging way to help students see their progress as they work toward understanding mathematics.

Remove the chart along the perforation and find a special spot to display it. You might choose a bulletin board, the refrigerator, a folder, or any other place that is easy to access and see.

After finishing each lesson, have your student color in or place a sticker over the corresponding circle on the chart. It is a great way to celebrate her hard work!

Daily Review Tracker

The *Daily Review Tracker* can be found on page 7 of the activity book.



This *Daily Review Tracker* is a tool for you to use with your student during the review section of each lesson. It helps build a strong foundation in mathematics by supporting concept retention and reinforcing understanding, while also tracking mastery of each skill.

Starting in Lesson 4, you will be prompted to enter the date next to skills that have been introduced. This will help you track which skills have been taught and should be included as part of your daily review.

In Lesson 5, you will begin using the tracker to identify areas where your student may need more practice to reach mastery. You will know she has achieved mastery when she can perform the skill consistently without assistance. Once she has demonstrated mastery, record the date in the ‘Date Mastered’ column.

As always, you are welcome to revisit any skill marked as mastered for a refresher or extra practice as needed.

Activity Sheets

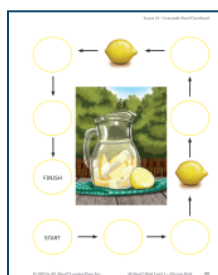
The activity sheets are highly motivating for most students, offering a variety of ways to practice the new concepts introduced in each lesson. They often include engaging themes, colorful visuals, and hands-on interactive elements that make learning both fun and meaningful.

Model	Total number of Logs	Total number of Logs	Date Mastered	Date Mastered
	4	3	$\frac{3}{4}$	$\frac{3}{4}$
		5		$\frac{5}{8}$
	3	2	$\frac{1}{3}$	
	6			$\frac{5}{8}$
		2		$\frac{2}{2}$
	4	3		

Take a look at the activity called “Charting the Logs” on page 51 of the activity book. When you get to Lesson 6, your student will take a look at a chart that has some missing information. She will use the given information to help the logger fill in what is missing and complete the chart.

Darby’s Math Fun!

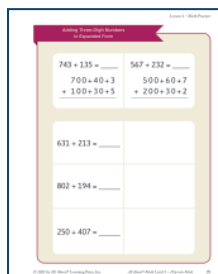
Math games make learning math exciting by turning practice into play, allowing students to explore concepts in a fun and interactive way. They will build confidence and fluency with math concepts while keeping your student engaged and motivated.



Darby’s Math Fun! can be found at the end of each new concept lesson. You can choose to play these games directly after the lesson or at another time. These games are designed to be played multiple times and are a great way to practice skills that are still developing.

Remove the Darby’s Math Fun! games along the perforation. Once you have completed a game, place it in a safe spot or folder for easy access for later play.

Math Practice



Starting in Lesson 4, your student will complete short Math Practice pages to review and reinforce what they have already learned. These pages include no more than six problems and are designed to be quick and focused. Math Practice pages allow your student to keep important skills fresh without feeling overwhelmed, helping build confidence and strengthen understanding over time.

Story Problems and Situations



Story Problem and Situation pages give your student a chance to engage with real-world math in a meaningful way. These pages list story problems or situations used throughout the lesson, allowing your student to read along or read aloud. She can use the page to underline important numbers, highlight the question being asked, and mark anything else that helps her make sense of the problem.

Certificate of Achievement

The *Certificate of Achievement* can be found on page 385 of the activity book.



Presenting your student with a certificate upon completing the Level 3 program is a wonderful way to celebrate her hard work and achievements. It will boost her confidence and give her a sense of pride in reaching an important milestone.



Learn about the Manipulatives

We will be using several types of manipulatives. Below is an introduction to some of their uses.

Connecting Cubes can be snapped together to form longer chains or structures. They can be used for:

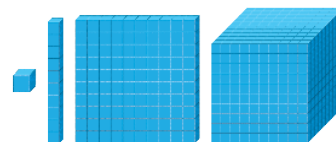


- **Multiplication and Division:** Connecting cubes can be used to model multiplication and division by creating equal groups or arrays. The cubes make it easier to see how multiplication works as repeated addition and how division works as sharing or grouping.
- **Fractions:** Different colored connecting cubes can show parts of a whole, compare fractions, or build fraction bars.

Two-Color Counters are small, circular discs that are red on one side and yellow on the other side. They can be used for basic addition and subtraction, making equal groups and arrays to demonstrate multiplication and division concepts, and game markers.



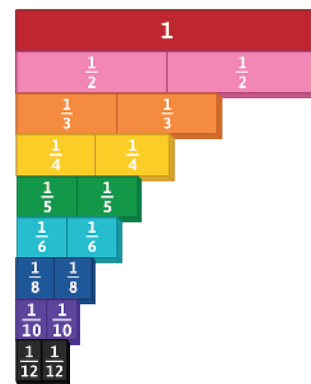
Base-10 Blocks also known as **place value blocks**, represent units of 1, 10, 100, and 1000. They can be used for:



- **Understanding Place Value:** Base-10 blocks are commonly used to help understand place value. Students can see how numbers and decimals are made by grouping the blocks together. Each type of block represents a different place value:
 - **Unit (ones):** Small cubes that represent the number 1 or 0.01.
 - **Rod (tens):** Long rods that represent groups of 10 or 0.1.
 - **Flat (hundreds):** Square flats that represent groups of 100 or one whole.
 - **Cube (thousands):** A cube that represents a group of 1000.
- **Multiplication and Division:** Using base-10 blocks helps students visualize the process of multiplication and division. Students can build arrays with the blocks to show how many groups there are and how many are in each group. The blocks help students break apart larger numbers into equal groups for division.

Fraction Tiles are colorful, rectangular pieces of different sizes that visually represent how parts make up a whole. They can be used for:

- **Reading and Writing Fractions:** Fraction tiles provide a visual representation of fractions as parts of a whole. For example, students can use the tiles to see that one out of four equal parts is written as $\frac{1}{4}$ and read as “one-fourth.”
- **Comparing Fractions:** Fraction tiles help students visually compare the sizes of different fractions. By lining up tiles representing parts of a whole, they can easily see which fractions are larger, smaller, or equal. For example, placing a $\frac{1}{2}$ tile above two $\frac{1}{4}$ tiles shows they are the same size.



Counting Bears are colorful, plastic bear-shaped manipulatives that can be used for game markers.

Ruler a straight tool with two sides: one marked in centimeters and the other in inches. While it's used to measure length and draw straight lines, it can also help students understand fractions. The inch side is divided into equal parts, like halves, fourths, and eighths, making it a useful tool for showing how fractions work on a number line.



The *All About Math* Manipulatives Kit also includes:

- **Dry Erase Pocket:** a transparent-plastic pocket that turns any Math Tool into a dry-erase board.

The Protractor will be used in higher levels.

The *All About Math* Manipulatives Kit comes in a plastic Storage Bin for ease and convenience.

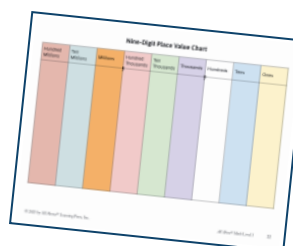




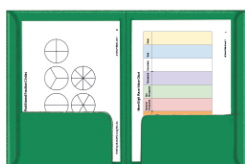
Preview *Darby's Math Tools*

Math Tools are printed resources that can be used in numerous ways to support student learning. In Level 3, Math Tools are primarily used to support operations and algebraic thinking. These tools include:

- Six-Digit Place Value Chart
- Place Value Chart (Hundreds)
- Place Value Disks
- Fraction Shapes
- Partitioned Fraction Circles
- 2D and 3D Shape Cards
- Fraction Model Cards
- Open Number Line
- Comparing Fractions Mat
- Number Cards
- Modeling Division and Multiplication Mat
- Multiplication Table
- Nine-Digit Place Value Chart



Math Tools are reused for many lessons, so once you use them, be sure to save them for future use.



Consider keeping the materials in a folder and storing them in a binder or in the manipulatives storage bin.



How Much Time Should I Spend on Math?

All About Math lessons are designed so that you can work at your student's pace. Here are some general guidelines.



Spend 20 minutes per day teaching math.

We recommend spending about 20 minutes per day, five days a week, on math instruction, but you can adjust this to meet your particular student's needs.

It can be helpful to set a timer. When 20 minutes are up, consider whether you have reached a logical stopping point in the lesson; you may want to complete the task or part of the task before stopping. Then, mark the spot in the lesson where you stopped. If your student is still engaged at the end of 20 minutes, feel free to extend the time if you wish.

When you begin teaching the next day, start with one or two items from the *Daily Review Tracker*, briefly review the New Teaching from the previous day, and then pick up in the teacher's manual where you left off previously. If your student struggles to remember previous learning, you can begin from an earlier point in the lesson.

Short daily lessons are much more effective than longer, less frequent lessons. Your student's attention is less likely to wander, and you can accomplish more when your student is actively engaged in the lesson.

If you aren't done with the lesson when the 20 minutes are up, don't worry! The next tip is for you.



Lessons often take more than one day to complete.

Please know that the lessons in *All About Math* are **not** meant to be completed in one day.

A number of variables, including your student's age, attention span, prior experience, the difficulty of the concept being taught, and the length of the lesson, all play a part in how quickly a lesson can be completed.

Teaching your student can be a wonderful way to show him that he has great value in your eyes. You can view this as an opportunity to build him up and help him develop skills and character. Can you see yourself as a calm, uncritical coach with the worthy goal of helping this child fulfill his natural potential? Imagine the type of teacher you would want: friendly, supportive, with a you-can-do-it attitude. Smile. Point out what your student has done right more often than you point out his mistakes. Treat lesson time as a special time between the two of you.

2

Complete Step-by-Step Lesson Plans

Hi!

I'm Darby! Darby, the dolphin. And I'll be popping up here and there throughout all your lessons in Level 3.

You're going to learn many new and exciting things about fractions, multiplying, and dividing. That's stuff I deal with every day. Does that sound odd coming from a dolphin?

Ha ha - actually it's not!

You see, when I leap out of the ocean and dive back in, everything changes! Fish dart away, new schools of fish cavort around and join up, other sea life float by... If I had to keep track of it all - I'd be multiplying, dividing, and counting fractions all day!



Lesson 6 Creating Models to Represent Fractions

Objective

Your student will learn to create a model to represent a fraction.

You Will Need

- ☐ *Flip into Math!* pages 49 to 52
- ☐ dry-erase board, pocket, and marker
- ☐ Fraction Shapes, *Darby's Math Tools*
- ☐ 2D and 3D Shape Cards, *Darby's Math Tools* page 8
- ☐ two pencils

Before You Begin

Preview Creating Models to Represent Fractions

In the previous lesson, your student identified fraction models that represented a given fraction. In this lesson, he will use that understanding to create his own models to represent fractions. He will explore how to create a model for a fraction by shading in unit fractions while using his understanding of numerators and denominators.

Understanding how to create models for fractions can help deepen the understanding of what fractions are so he can apply that knowledge to everyday situations.

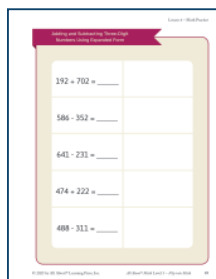
Review

Date	Topic	Review Questions	Answers
		1. Add $\frac{1}{2}$ and $\frac{1}{4}$.	$\frac{3}{4}$
		2. Subtract $\frac{1}{4}$ from $\frac{1}{2}$.	$\frac{1}{4}$
		3. Multiply $\frac{1}{2}$ by $\frac{1}{4}$.	$\frac{1}{8}$
		4. Divide $\frac{1}{2}$ by $\frac{1}{4}$.	2
		5. A box contains 100 marbles. If $\frac{1}{4}$ of the marbles are red, how many red marbles are there?	25
		6. A box contains 100 marbles. If $\frac{1}{4}$ of the marbles are red, how many blue marbles are there?	75
		7. A box contains 100 marbles. If $\frac{1}{4}$ of the marbles are red, how many green marbles are there?	75
		8. A box contains 100 marbles. If $\frac{1}{4}$ of the marbles are red, how many yellow marbles are there?	75
		9. A box contains 100 marbles. If $\frac{1}{4}$ of the marbles are red, how many purple marbles are there?	75
		10. A box contains 100 marbles. If $\frac{1}{4}$ of the marbles are red, how many white marbles are there?	75

Daily Review

As part of your math time each day, refer to your student's *Daily Review Tracker*. Choose one or two skills, and take a few minutes to practice.

Review (continued)



Math Practice

Turn to page 49 in the *Flip into Math!* activity book.

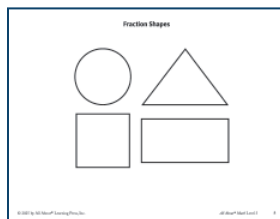
“Let’s review adding and subtracting two three-digit numbers by using expanded form. You will rewrite each problem in expanded form in the workspace and then solve the problem. Pay close attention to the symbol so that you know if you need to add or subtract for each problem.”

Answer Key

- $800 + 90 + 4 = 894$
- $200 + 30 + 4 = 234$
- $400 + 10 + 0 = 410$
- $600 + 90 + 6 = 696$
- $100 + 70 + 7 = 177$

Identifying Fraction Models

“We have learned how to partition shapes and to identify the fractions that the models represent. What is important about partitioning shapes?” *The parts need to be equal.*



Fraction Shapes

Find the *Fraction Shapes* page from *Darby's Math Tools*. Place it into the dry-erase pocket.

“Partition the square into three parts and the rectangle into four parts.” Shade in two parts of the square and one part of the rectangle.



“Let’s pretend that these shapes are garden beds that Henry and his dad are going to plant in their backyard. Henry’s dad divided a garden bed into four equally sized sections to plant tomatoes, squash, peppers, and cucumbers. Find the model of the garden bed that will be used for growing the vegetables, and then write the fraction that represents the model beside it.” (*Answers: the rectangle, $\frac{1}{4}$*)

Review (continued)

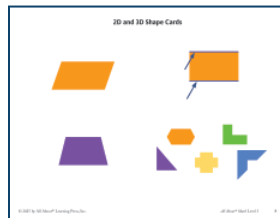
Clear the lines and shading on the square, but do not erase the fraction or the model on the rectangle so that it can be used next in the lesson.

“The rectangle is divided into four parts, and one part is shaded in, so the fraction is $\frac{1}{4}$. We will use identifying and partitioning to help us create models.”

New Teaching

Identifying Shapes

“Before we dive into our main lesson, let’s explore some 2D shapes and their attributes.”



2D and 3D Shape Cards

Find the *2D and 3D Shape Cards* from *Darby’s Math Tools* on page 8. You will only need the cards for: parallel, parallelogram, and trapezoid. Place the other cards to the side.

Place the shape card showing parallel in the workspace.

“Let’s look a little closer at the sides that make up this quadrilateral. Notice there are arrows pointing to the opposite sides. When sides, or lines, are the same distance apart, we say they are parallel.”

Place two pencils in the workspace. Make sure the pencils are an equal distance apart to demonstrate parallel.

“We can imagine that these pencils are two lines. As long as they are an equal distance apart, we can say they are parallel. Even if the lines continue on past the ends of the pencils, they would still never touch because they are parallel.”

Angle one pencil so that the pencils are no longer an equal distance apart.

“If I move one pencil, the pencils are no longer an equal distance apart, so they are not parallel. Are the shorter sides (point to the shorter sides) on this rectangle parallel, and how do you know?” *Yes, they are parallel because they are the same distance apart.*

Keep the parallel card in the workspace. Place the shape card showing a parallelogram in the workspace.

New Teaching (continued)

“What do you notice about this shape?” Answers will vary. *It has four sides and four vertices. It is a quadrilateral. The opposite sides are parallel.*

“Great observations. This shape is a quadrilateral because it has four sides. We can be more specific when we identify quadrilaterals by looking at the sides. A shape that has both pairs of opposite sides (point to the opposite sides) that are parallel and equal in length is called a parallelogram.”

“Let’s look at the first shape card we started with, showing parallel. What are the different ways we can identify this shape?” *It can be called a quadrilateral or a rectangle.*

“Right. Could it also be called a parallelogram, and how do you know?” *Yes, because the opposite sides are parallel and equal.*

Keep the parallel and parallelogram cards in the workspace. Place the shape card showing a trapezoid in the workspace.

“Now, let’s look at one more shape. Can this shape be called a parallelogram, and how do you know?” *No, it can not be called a parallelogram because one set of the opposite sides are not parallel.*

“You are right. This shape can be called a quadrilateral because it has four sides, but it is not a parallelogram. Point to the sides that are not parallel.” *(Answer: the sides on the left and right)*

“The top and bottom (point to the sides) are parallel, but the left and right sides (point to the sides) are not parallel. We call this shape a trapezoid. A trapezoid has only one set of parallel sides.”

Bring this learning into the real world by asking your student to find examples of these shapes and attributes in his environment. Connecting shapes to real-world objects helps deepen understanding.

Include these cards as part of your daily review.

Creating Models to Represent Fractions

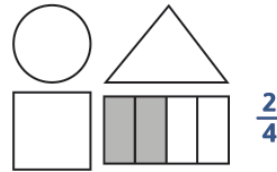
“We are going to imagine a logger at work. A logger is someone who cuts down trees and then cuts the trees into smaller pieces to sell. This logger cuts the logs into equal parts to make them easier to stack and sell.”

“Now, let’s pretend that this rectangle (point to the rectangle on the *Fraction Shapes* page that was previously used) is a log that has been cut by a logger into four equally sized parts. What is the unit fraction for this model?” *One-fourth.*

New Teaching

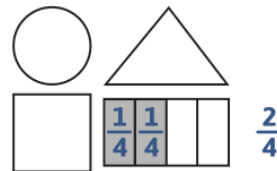
(continued)

“Now, the logger needs to cut two-fourths of a whole tree and load it onto his truck to sell. Each part is one-fourth, so two-fourths means two unit fractions of one-fourth each.” Shade in one more unit fraction in the model.



“I shaded in one more unit fraction to represent two-fourths. The numerator changed from one to two because we now have two parts of the model shaded in. Let’s label and count the unit fractions.”

Write the unit fraction $\frac{1}{4}$ in each of the shaded parts. Point to each unit fraction as you count with your student. “One-fourth, two-fourths.”



Clear the *Fraction Shapes* page.

Partition the square on the *Fraction Shapes* page into thirds. “How is this shape partitioned, and what is the unit fraction?” *It is partitioned into thirds, so the unit fraction is one-third.*



Write the fraction $\frac{2}{3}$ beside the square. “Now, the logger needs to stack two-thirds of a log. How many unit fractions do you need to shade in to represent the fraction?” *Two.*

“So, two-thirds is made up of two unit fractions of one-third each. Shade in the model to represent the fraction two-thirds.”



New Teaching (continued)

“When we need to create a model to represent a fraction, we can look at the denominator to see how many parts the model will be partitioned into and then look at the numerator to see how many parts need to be shaded in.”

Write the fraction $\frac{3}{6}$ on the dry-erase board. “What is the unit fraction in this model?” *One-sixth.*

“Good. That means each of the six parts is a unit fraction of one-sixth. Use the *Fraction Shapes* page to represent the fraction three-sixths. You can use any of the shapes on the page, but think about which shape would be the best to partition into sixths.” Your student should partition a shape to create six equally sized parts and then shade in three of the parts.

Guide your student to choose a shape that can easily be divided into six equally sized parts. Remind him that each part should be the same size. For example, the triangle would not be the best shape to use to represent $\frac{3}{6}$ because the parts would not be equal.



“Good. The denominator lets you know to partition the shape into six parts, and the numerator lets you know to shade in three parts to represent the fraction.”

Continue using the same steps to create a model that represents the following fractions:

$$\frac{5}{8}, \frac{1}{2}, \frac{2}{3}, \frac{4}{4}$$

Complete Activity Sheet

“Let’s create models to represent fractions.”

Model	Number of Equal Parts	Number of Parts Shaded	Fraction	Number Sold
	4	3	$\frac{3}{4}$	$\frac{3}{4}$
	5	5		$\frac{5}{8}$
	3	2	$\frac{1}{3}$	
	6			$\frac{5}{6}$
	2			$\frac{2}{2}$
	6	3		

Charting the Logs

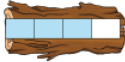


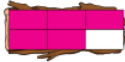


Turn to page 51 in the *Flip into Math!* activity book.

“Each log has been cut, and some parts of the log have been sold. Help the logger fill in the missing information.”

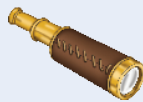
New Teaching (continued)

Answer Key

Lesson 6 – Charting the Logs

Model	Total Number of Equal Parts	Total Number of Parts Sold	Unit Fraction	Fraction Sold
	4	3	$\frac{1}{4}$	$\frac{3}{4}$
	8	5	$\frac{1}{8}$	$\frac{5}{8}$
	3	2	$\frac{1}{3}$	$\frac{2}{3}$
	6	5	$\frac{1}{6}$	$\frac{5}{6}$
	2	2	$\frac{1}{2}$	$\frac{2}{2}$
	6	3	$\frac{1}{6}$	$\frac{3}{6}$

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Look For

Your student might see the shaded parts as the numerator and the unshaded parts as the denominator and write that as a fraction. For example, if he see five parts shaded out of eight, he might write the fraction as $\frac{5}{3}$.

Here's How to Help: It may be helpful for your student to start by identifying the unit fraction for each log before trying to determine the part that is cut or sold. You can have your student first look at how many parts the log was cut into and then ask him to tell you what fraction one of those parts represents.

Math Reflection

“Let’s Reflect!”

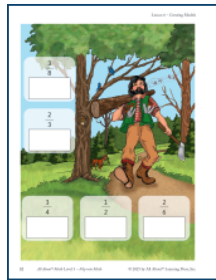
Ask some questions to guide your student's reflection:

- “How can we use unit fractions to create a model for a fraction?”
- “How do the numerator and the denominator of a fraction help you create a model?”

New Teaching (continued)

Extended Practice (Optional)

If your student is not able to create models to represent fractions, or he expressed the need for more practice, continue working on this skill.

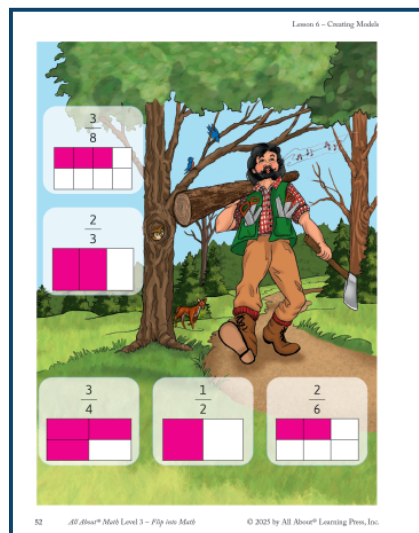


Creating Models

Turn to page 52 in the *Flip into Math!* activity book.

Have your student represent each fraction by partitioning the model and then shading in the parts.

Answer Key



Your student will continue to practice creating models to represent fractions in future lessons. You can proceed to the next lesson without full mastery of this skill.

Logger's Secret Fractions

"Let's practice our math skills!"

Materials

- Logger's Secret Fractions Gameboard, *Flip into Math!* pages 53 to 55
- Logger's Secret Unit Fraction Cards, *Flip into Math!* page 57
- Logger's Secret Fraction Cards, *Flip into Math!* pages 59 to 60
- divider (such as an open hardcover book)

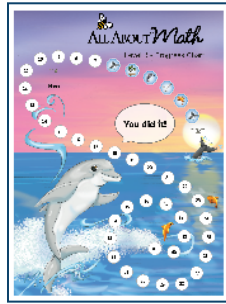
Directions

1. Each player gets a *Logger's Secret Fractions* gameboard on which to create the models of the secret fractions.
2. Cut apart the fraction cards, mix them up, and then place them face down in a stack.
3. Cut apart the unit fraction cards, mix them up, and then place them face down in a separate stack.
4. Each player draws four secret fraction cards, looks at them, and then places them face down next to each of the logs on the gameboard. These have the fractions that the players will try to make with the unit fraction cards.
5. The players take turns. For their turn, a player can either pick up one unit fraction card or trade one secret fraction card for a new secret fraction card from the stack.
6. The players continue to draw and line up unit fraction cards on top of the logs to create their secret fractions. For example, if they have $\frac{3}{4}$ as a secret fraction next to the first log, they would line up the $\frac{1}{4}$ unit fractions on top of that log until they complete that fraction.
7. If the unit fraction card drawn does not match the secret fraction, the player can place the unit fraction card to the side and save it for later use. If a player trades a secret fraction card, he can immediately place any unit fraction cards he had set aside.
8. When two secret fractions have been created, the player will reveal both secret fractions he created and have the other player check.
9. The first player to make two secret fractions correctly is the winner!



Track Your Progress

Mark the Progress Chart



Have your student mark Lesson 6 on the Progress Chart.

Telling stories is so much fun.
Don't you agree? Because with every
story you learn something new!

Here's a story for you: Did you know that
I have my very own whistle sound? I do! And
as I was swimming around with half of my six
friends today, I used it to call the other half.
How many friends did I whistle for?

(Did you guess 3? You're so smart!)



Lesson 10 Equivalent or Not?

Objective

Your student will learn to identify equivalent fractions.

You Will Need

- ☐ *Flip into Math!* pages 89 to 92
- ☐ Fraction Tiles
- ☐ Partitioned Fraction Circles, *Darby's Math Tools*
- ☐ dry-erase board, pocket, and marker
- ☐ 2D and 3D Shape Cards, *Darby's Math Tools* page 9
- ☐ Base-10 Block: ones unit or thousands cube

Math Vocabulary

equivalent fractions

Before You Begin

Preview Fraction Equivalency

In this lesson, your student will use representations to determine if two fractions are equivalent. He will not need to know if a fraction is more or less than another but only if both are the same amount. In previous lessons, he worked to represent fractions by using models and number lines. These tools will be important when determining if two fractions are equivalent.

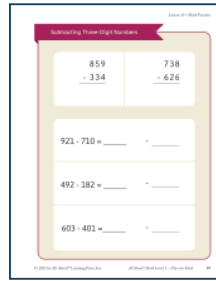
Learning this skill is important in developing the skill of comparing fractions in the next few lessons.

Review

Daily Review

As part of your math time each day, refer to your student's *Daily Review Tracker*. Choose one or two skills, and take a few minutes to practice.

Review (continued)



Math Practice

Turn to page 89 in the *Flip into Math!* activity book.

“Let’s review subtracting two three-digit numbers by using the standard algorithm. Two problems are already stacked for you to subtract. You will have to rewrite the other problems before subtracting.”

Answer Key

- 525
- 112
- 211
- 310
- 202

Review Representing Fractions

“How can we represent fractions?” *By using models, number lines, and fraction tiles.*

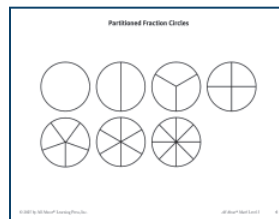
“Yes. Let’s represent the fraction $\frac{2}{4}$ in the different ways. Draw a number line to represent fourths on the dry-erase board. Then label each tick mark.”



“Good. Point to the tick mark that shows $\frac{2}{4}$. I will place a dot on the tick mark to represent $\frac{2}{4}$ on the number line.”



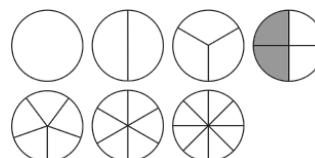
Do not erase the number line. It will be used later in the lesson.



Partitioned Fraction Circles

Find the *Partitioned Fraction Circles* from *Darby’s Math Tools*. Place this page into the dry-erase pocket.

“You showed the fraction $\frac{2}{4}$ on a number line. Now, let’s use a model to represent $\frac{2}{4}$. Choose a circle, and then shade it to represent $\frac{2}{4}$.”



Review (continued)

“Good. Now, use the fraction tiles to represent $\frac{2}{4}$.”

Your student should show two tiles with a unit fraction of $\frac{1}{4}$.



“You represented $\frac{2}{4}$ by using a number line, a model, and the fraction tiles! Nice work!”

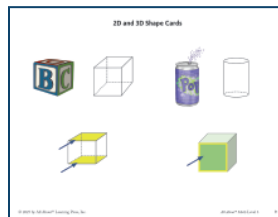
Keep all the fraction representations of $\frac{2}{4}$ for later in this lesson.



New Teaching

Identifying Shapes

“Before we dive into our main lesson, let’s explore some 3D shapes and their attributes.”



2D and 3D Shape Cards

Find the *2D and 3D Shape Cards* from *Darby’s Math Tools* on page 9. You will only need the cards for: base, face, cube, and cylinder.

“Let’s focus on 3D shapes and their attributes. Three-dimensional shapes are solid figures that have length, width, and height (or depth). Unlike flat 2D shapes, 3D shapes take up space and can be held, turned, and viewed from different angles.”

Place the shape card showing the cube in the workspace.

“Where have you seen something that looks similar to this shape?”
Answers will vary. It looks like a manipulative we have used before.

Place a ones unit or the thousands cube in the workspace.

“That is right. We used the ones unit and the thousands cube to help us learn about place value. This shape is called a cube. What do you notice about the sides of the cube?” *I notice all the sides look like squares.*

New Teaching

(continued)

“That is a characteristic of a cube. All the sides are square and equal. When we talk about the sides of a 3D shape, we use a different word.”

Place the shape card showing face in the workspace.

“Each side that makes up a 3D shape is called a face of the shape (point to the face of the cube on the card). Faces are flat surfaces that make up the outside of the 3D shape and are shaped like 2D polygons. How many faces does the cube have?” *Six.*

Place the shape card showing base in the workspace.

“Right. When we put a 3D shape down on a surface, it will be sitting on one of the faces. We call that the base of the shape. Sometimes the top of the shape can also be considered a base if it matches the bottom in shape and size.”

Place the shape card showing the cylinder in the workspace.

“Use the attributes we have learned and tell me what you notice about this three-dimensional shape. (Point to the cylinder.)” *I see that it has a base that looks like a circle on the top and bottom, and a curved surface.*

“Good. A shape with two circular bases and one curved surface is called a cylinder.”

Bring this learning into the real world by asking your student to find examples of these shapes and attributes in his environment. Connecting shapes to real-world objects helps deepen understanding.

Learning about Equivalent Fractions

“Let’s imagine that a group of friends is making muffins for a party. The recipe asks for $\frac{2}{4}$ of a cup of sugar. They can find only a $\frac{1}{2}$ cup to measure the sugar. Let’s help them figure out if this is more sugar, less sugar, or the right amount of sugar they need for the recipe. You already represented the fraction $\frac{2}{4}$ in different ways. Let’s represent the fraction $\frac{1}{2}$, too.”

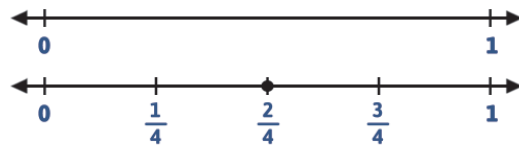
In this lesson, as you are comparing fractions with your student to determine if they are equivalent, it is important to use the same type of model for each comparison. If he is comparing by using number lines, for example, both number lines should be exactly the same size. If he is using models, they should be the same shape and size.



New Teaching

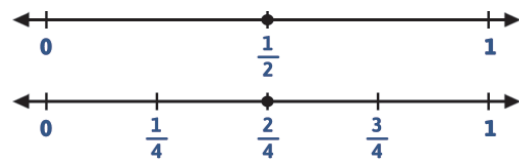
(continued)

On the dry-erase board, draw another number line directly above or below the number line representing $\frac{2}{4}$. Have the new number line also start at zero and end at one.



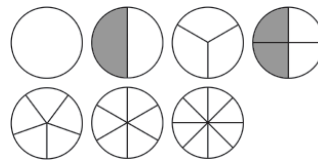
“Here is another number line that starts at zero and ends at one. It is the same size as the number line you drew to represent $\frac{2}{4}$. Show me how you can represent the fraction $\frac{1}{2}$ on this number line.”

Your student should draw one tick mark in the middle of the number line, label it $\frac{1}{2}$, and add a point.



“Good work representing $\frac{1}{2}$ on the number line. The number lines are exactly the same length, which will help us compare the fractions. I see that $\frac{2}{4}$ and $\frac{1}{2}$ are in the same place on each number line.”

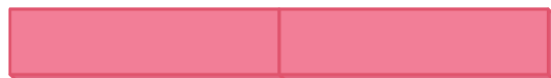
“Now, let’s represent $\frac{1}{2}$ by using the *Partitioned Fraction Circles*. Find the circle that you will use to represent $\frac{1}{2}$, and then shade it.”



“Great job! You represented $\frac{1}{2}$ with a model. What do you notice about this model and the model that you shaded for $\frac{2}{4}$?” *They look like they are shaded the same amount.*

“I agree! The sizes of the shaded parts look the same. I also noticed that the sizes of the circles are exactly the same.”

“Now, represent $\frac{1}{2}$ by using the fraction tiles. Find the $\frac{1}{2}$ fraction tiles, and place them face down.”



New Teaching

(continued)

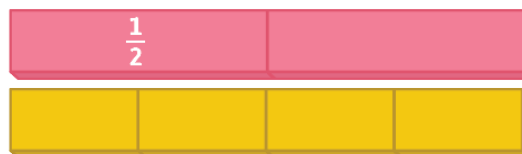
When using fraction tiles to compare fractions, it's important to show all the unit fraction tiles that make up a whole because it will help your student clearly see how many parts each fraction has and how those parts relate to each other. This visual support makes it easier to compare sizes, recognize equivalent fractions, and understand that smaller denominators mean larger parts.

Tip!

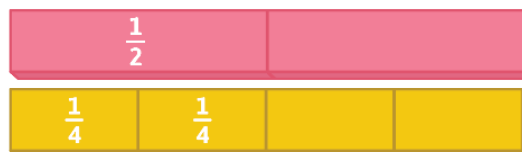
Point to the fraction tiles. “These tiles represent one whole. When I flip over one of the tiles, the tiles will represent one-half of a whole. One of the two parts is flipped over.”



“Now, let's do the same steps using the tile that show the unit fraction $\frac{1}{4}$. Find the tiles, and place them under the $\frac{1}{2}$ tiles, face down.”



“Good. You represented another one whole that is the same length as the one whole using the $\frac{1}{2}$ tiles.” Flip over two $\frac{1}{4}$ tiles.



“Take a look at the $\frac{1}{2}$ tile and $\frac{1}{4}$ tiles. What do you notice?” *They are the same size.*

“You used a number line, a model, and fraction tiles to represent $\frac{1}{2}$. With all the representations, it looks like $\frac{2}{4}$ and $\frac{1}{2}$ show the same amount. It is important that the number lines are the same length and that the fraction models are exactly the same shape and size to show this. The friends need $\frac{2}{4}$ of a cup of sugar. Can they use the $\frac{1}{2}$ cup?” *Yes, they can because they are the same size.*

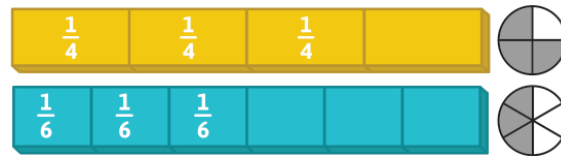
“That's right! $\frac{2}{4}$ and $\frac{1}{2}$ are the same size. When two fractions represent the same amount, they are called **equivalent fractions**.”

New Teaching (continued)

Are They Equivalent?

“The friends are eating some muffins that they baked. Polly ate $\frac{3}{4}$ of a muffin. William ate $\frac{3}{6}$ of a muffin. They are wondering if they ate the same amount of their muffins. You can use number lines, models, or fraction tiles to help you find out if the fractions $\frac{3}{4}$ and $\frac{3}{6}$ are equivalent. Remember, equivalent means that they represent the same amount.”

Write the fractions $\frac{3}{4}$ and $\frac{3}{6}$ on the dry-erase board. Allow your student to choose the representation he is the most comfortable with. You can represent the fractions in another way to help support his answer. For example, if your student chooses fraction tiles, you can also represent the fractions by using models.



“Tell me about the representation you chose to use to find out if the fractions are equivalent.” *I chose to use fraction tiles. I used three one-fourth tiles and three one-sixth tiles.*

“Do you think the fractions are equivalent, and why?” *I do not think the fractions are equivalent because when I lined them up, I noticed that they were not the same size.*

“Great work! You represented each fraction and noticed that they were not the same size.”

Have your student determine if the following pairs of fractions are equivalent. Encourage him to use different representations if he chooses only one. For example, if your student likes to shade models, encourage him to try making number lines.

- $\frac{1}{2}$ and $\frac{1}{3}$
- $\frac{3}{4}$ and $\frac{6}{8}$
- $\frac{2}{3}$ and $\frac{2}{6}$

Complete Activity Sheet

“Let’s figure out if more fractions are equivalent!”

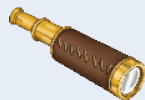
Muffin Equivalency

Turn to pages 91 and 92 in the *Flip into Math!* activity book.

Have your student write in the fraction that the muffin model represents. If the fraction is given and the model is blank, have your

student shade the model to represent the fraction. Your student should determine if the fractions are equivalent and then circle Yes or No.

Answer Key



Look For

Your student might try to determine if the fractions are equivalent just by looking at the numbers. For example, he might think that $\frac{4}{4}$ is more than $\frac{3}{3}$ because the number four has a larger value than the number three.

Here’s How to Help: Have your student use additional models, such as the number line or fraction tiles, to help him determine if the fractions are equivalent.

New Teaching (continued)

Math Reflection

“Let’s Reflect!”

Ask some questions to guide your student's reflection:

- “Do you like to use number lines, tiles, or shapes when representing fractions, and why?”
- “What do you look for when deciding if two fractions are equivalent?”
- “What is something that challenged you today?”

Extended Practice (Optional)

If your student is not able to recognize equivalent fractions, or he expressed the need for more practice, continue working on this skill.

“Liam and Lacey are eating pieces of brownies. They want to know if they are eating the same amount of each brownie.”

Have your student use two different methods (number lines, models, fraction tiles) to represent each fraction. Your student will use the second method of his choice to check the first representation. Then, have him determine if the pairs of fractions are equivalent:

- Liam: $\frac{3}{4}$, Lacey: $\frac{4}{4}$ (Answer: no)
- Liam: $\frac{1}{3}$, Lacey: $\frac{2}{6}$ (Answer: yes)
- Liam: $\frac{2}{2}$, Lacey: $\frac{3}{3}$ (Answer: yes)

Your student will continue practicing equivalent fractions in future lessons. You can proceed to the next lesson without full mastery of this skill.

Mark the Daily Review Tracker

Date	Skill	Standard	Status
	Comparing Fractions	3.NF.A.1	
	Adding and Subtracting Fractions	3.NF.A.2	
	Multiplying Fractions	3.NF.B.1	
	Dividing Fractions	3.NF.B.2	

Write today's date next to the skill: Comparing fractions. Include this in your rotation of daily review items.

Darby's Math Fun!

Who Took the Muffins?

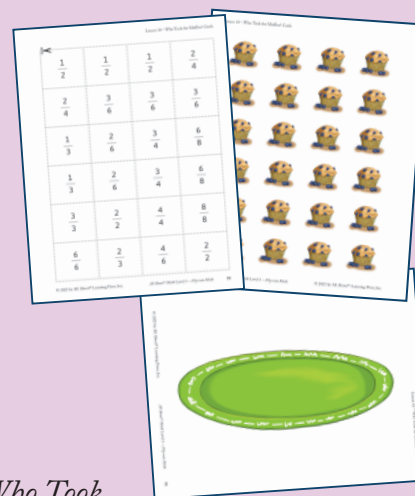
“Let’s practice our math skills!”

Materials

- Who Took the Muffins? Cards, *Flip into Math!* page 93
- Who Took the Muffins? Gameboard, *Flip into Math!* page 95
- Partitioned Fraction Circles, *Darby's Math Tools*
- Fraction Tiles
- dry-erase board and marker

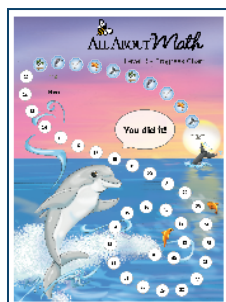
Directions

1. Cut apart the *Who Took the Muffins?* cards.
2. Mix up the cards, and then scatter them face down on the *Who Took the Muffins?* gameboard.
3. The players take turns turning over two muffins from the muffin plate and then building a model of the fractions on the cards by using a number line, fraction tiles, or a fraction model.
4. The player decides if the two fractions are equivalent. If they are equivalent, he keeps the fraction cards. If not, he turns both fraction cards back over.
5. If the fraction cards display the same fraction, the player can replace one card with a new card. The fractions must be different fractions.
6. The first player to take six muffins (three pairs of equivalent fractions) wins!
7. Optional: This game is a twist on the old song, “Who Took the Cookie from the Cookie Jar?” Players may chant “Who took the Muffin from the Muffin Plate?” as they play.



Track Your Progress

Mark the Progress Chart



Have your student mark Lesson 10 on the Progress Chart.

Lesson 24 Division as an Unknown-Factor Problem, Part 2

Objective

Your student will learn to relate multiplication and division and to recognize division as an unknown-factor problem.

You Will Need

- | | |
|--|--|
| <input type="checkbox"/> <i>Flip into Math!</i> pages 221 to 225 | <input type="checkbox"/> dry-erase board, pocket, and marker |
| <input type="checkbox"/> Connecting Cubes | <input type="checkbox"/> glue |
| <input type="checkbox"/> blank sheet of paper | <input type="checkbox"/> Two-Color Counters |

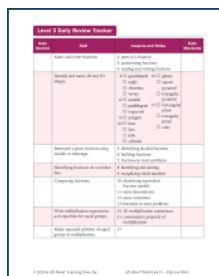
Before You Begin

Preview Relating Division Diagrams and Equations to Multiplication

In this lesson, your student will continue to explore the relationship between multiplication and division. In the previous lesson, he learned that the quotient in a division problem is the same as an unknown factor in a related multiplication problem. He used arrays to help identify this relationship. In this lesson, he will find the number of groups or how many are in each group in a division situation. He can also identify the unknown factor in a multiplication equation. Thinking about numbers flexibly helps him choose the best strategy, see the connection between multiplication and division, and solve problems with confidence.

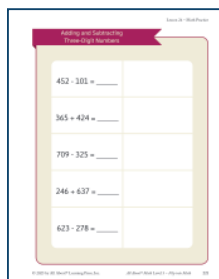
Understanding how multiplication and division are connected is important. These two operations work closely together because knowing a multiplication fact can help you figure out a related division fact. For example, if your student knows that $4 \times 6 = 24$, then he can also figure out that $24 \div 6 = 4$ or $24 \div 4 = 6$. Using multiplication to find a missing number in a division equation is a helpful strategy that will support him in future lessons. Strengthening this understanding now helps him become more accurate when solving equations.

Review



Daily Review

As part of your math time each day, refer to your student's *Daily Review Tracker*. Choose one or two skills, and take a few minutes to practice.



Math Practice

Turn to page 221 in the *Flip into Math!* activity book.

“Let’s review adding and subtracting three-digit numbers. Rewrite each problem by stacking the numbers in the standard algorithm. Then, add or subtract to find the answer.”

Answer Key

- 351
- 789
- 384
- 883
- 345

Relating Division and Multiplication Equations

Place a blank sheet of paper into the dry-erase pocket.

“Previously, you learned about division. When we divide, we split an amount into equal groups. One way to find the answer to a division equation is to use multiplication. What is the answer to a division equation called?” *It is called the quotient.*

“Right! If we have 15 buttons that we are sorting into 3 different jars, the number of buttons we place into each jar will be the quotient. How can we use multiplication to think about this situation?” *We can think about it as 3 groups of buttons that equals 15.*

“Yes. We can write both a multiplication and a division equation using the same numbers.”

Write the equation $12 \div 4 = \underline{\quad}$ on the dry-erase pocket.

Review (continued)

“Make an array to match this equation. Then, use your array to write a multiplication equation that will help us find the quotient.” (*Answer:* $4 \times \underline{\quad} = 12$, or $\underline{\quad} \times 4 = 12$)



“The blank space represents the quotient in the division equation and a factor in the multiplication equation.”

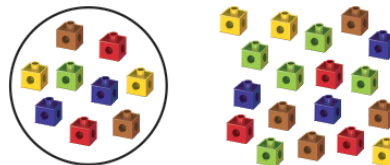
New Teaching

Relating Division Models and Equations to Multiplication Equations

“Let’s imagine that we are helping with the laundry today! We have 27 pieces of clothing to wash. Count out 27 connecting cubes that we’ll pretend are pieces of clothing.”

“We don’t know how many baskets there are for sorting the clothes, but we do know that there are nine pieces of clothing in each basket, or nine in each group. I can make a model to help represent this situation; I’ll start with making one group of nine.”

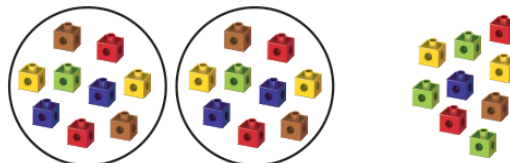
Pull nine cubes from the pile to form a separate group on the dry-erase pocket. Draw a circle around the nine connecting cubes.



Your student can connect the cubes as a tower if preferred, to help keep them organized.



“We still have clothes in a pile that are not in a basket (point to the unused connecting cubes), count out another group of nine.”

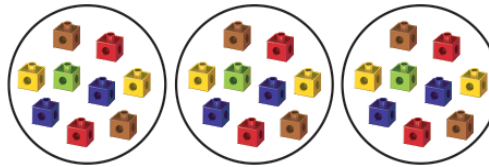


New Teaching

(continued)

“It looks like there are more clothes that we haven’t placed in a basket. What can we do?” *We can count out another group of nine.*

“Right! Go ahead and make another group of nine.”



“We’ve put all the clothes into baskets. In this example, we started with 27 pieces of clothing and divided them into equal groups.” Write $27 \div$ on the dry-erase board.

“We didn’t know how many groups there were, and you can show this by dividing by an unknown amount. An unknown amount is the number that we are trying to find. We can use a blank space to show the unknown amount in our equation.” Continue writing from $27 \div$ so that it reads $27 \div \underline{\hspace{1cm}}$.

“Let’s complete the equation with what we did know: there were 9 pieces of clothing and we divided them into equal groups.” Write $= 9$ to finish the equation on the dry-erase board.

$$27 \div \underline{\hspace{1cm}} = 9$$

“Let’s use your model to solve for the unknown amount. How many groups do you have?” *Three.*

“Right! So, $27 \div 3 = 9$ (write 3 on the blank space in the equation). This division equation represents our situation. However, we can also use multiplication to solve this problem. Take a look at the model that you made with the connecting cubes. We have some groups of 9. We can write this as an unknown amount times 9.”

Write $\underline{\hspace{1cm}} \times 9$ under the division equation on the dry-erase board.

“Some groups of 9 equals a total of 27.” On the dry-erase board, after $\underline{\hspace{1cm}} \times 9$, write $= 27$ so that the equation reads $\underline{\hspace{1cm}} \times 9 = 27$.

$$27 \div \underline{3} = 9$$
$$\underline{\hspace{1cm}} \times 9 = 27$$

“How many groups of nine does your model show?” *Three.*

New Teaching

(continued)

“Yes! So, $3 \times 9 = 27$ (write 3 on the blank space in the equation). The unknown amount is the same in both equations! Let’s look at another example.”

Clear the dry-erase board and the dry-erase pocket. Place 54 connecting cubes to the side of the dry-erase pocket.

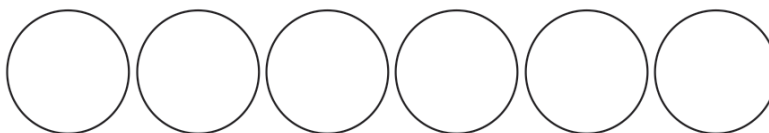
“In division, we can divide a number into groups, or we can divide a number by how many items are in each group. Last time, we found how many groups there were. This time, we’ll find how many items each group has. Here are more pieces of laundry. How many are there in all?” *There are fifty-four.*

“Right! We’ll divide the pieces of laundry by an unknown amount again, but this time, the unknown amount represents how many pieces of laundry will be placed into each basket.”

Write $54 \div \underline{\hspace{1cm}} = 6$ on the dry-erase board.

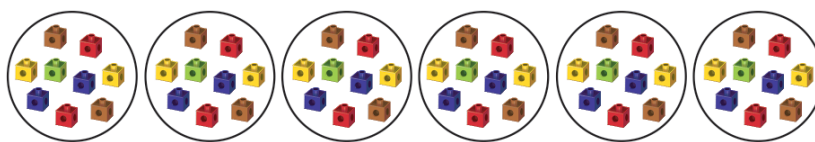
“The quotient (point to the 6) tells us there are 6 baskets, or 6 groups. We can use the connecting cubes to model this by making 6 equal groups with the 54 connecting cubes.”

“Since we know how many groups we have this time, I will draw a circle to represent each of the six groups. Each group represents one basket of laundry.”



“We’ll add one connecting cube, or one piece of laundry, one at a time, to each group to make sure that we create equal groups.” Have your student add one connecting cube at a time to each group.

“Now, we’ve divided the 54 pieces of laundry into 6 baskets.”



“Our model also shows multiplication! We knew we had six baskets, but we didn’t know how many pieces of laundry were in each basket.” Write $6 \times \underline{\hspace{1cm}}$ under the division equation on the dry-erase board.

New Teaching

(continued)

“Six groups of an unknown amount is the same as 54 because we had 54 total pieces of laundry.” After $6 \times \underline{\quad}$, write $= 54$ so that the equation reads $6 \times \underline{\quad} = 54$.

“The unknown amount in each equation represents the number of pieces of laundry in each basket, or the number of connecting cubes in each group. What is the unknown amount in each equation?” *Nine.*

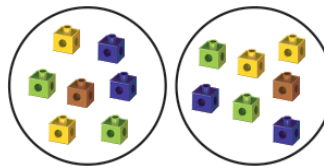
“Right. Write a nine on the blank space in each equation on the dry-erase board.”

$$54 \div \underline{9} = 6$$

$$6 \times \underline{9} = 54$$

“After the clothes dry, we’ll sort them into baskets to put away. This time, let’s imagine that we have 14 pieces of clothing. We have some baskets to put the clothes into, and each basket can hold 7 pieces of clothing.”

“Make a model to represent this situation.”



“Nice job! Now, let’s write a division equation to match.” Write a division equation with blank spaces on the dry-erase board.

$$\underline{\quad} \div \underline{\quad} = \underline{\quad}$$

Have your student fill in the blank spaces on the dry-erase board, creating the equation $14 \div 2 = 7$.

If your student is unsure, reread the situation and have him look at his model. Have him fill in each blank by asking how many he started with, how many groups there are, and how many are in each group.



“Nice work!”

Write the equations $2 \times 7 = 14$ and $2 \times 14 = 7$ on the dry-erase board.

“Which of these two equations represents the same situation with multiplication? How do you know?” *The equation $2 \times 7 = 14$ because*

New Teaching (continued)

there are 2 baskets that each have 7 clothing items, not 2 baskets that each have 14 clothing items. The number 14 represents the total number of pieces of clothing, so it is the product.

“When looking at a division equation or situation, how do you know which number is the same as the product in a related multiplication equation?” *It’s the total number of items, or the number you start with, in a division equation.*

“To find the number of groups or the amount in each group when dividing, do you look at the factor or the product in a multiplication equation?” *The factor.*

Use the following situations to continue relating multiplication to division. Have your student recognize that the unknown value in each division equation is the same as an unknown factor in a multiplication equation. Have him create a model using the connecting cubes to help him find and write the corresponding division and multiplication equations, solving for the unknown value.

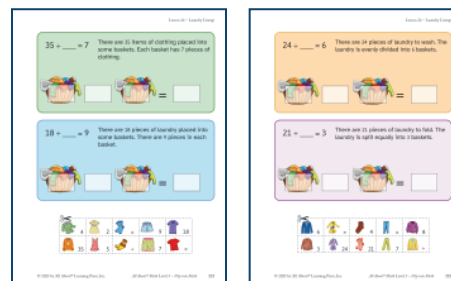
- “There are 63 pieces of laundry. There are 7 pieces of laundry in each basket.” (*Answers: $63 \div \underline{\hspace{1cm}} = 7$ and $\underline{\hspace{1cm}} \times 7 = 63$, $63 \div 9 = 7$, $9 \times 7 = 63$*)
- “There are 56 pieces of laundry. The laundry is in 8 baskets.” (*Answers: $56 \div \underline{\hspace{1cm}} = 8$ and $8 \times \underline{\hspace{1cm}} = 56$, $56 \div 7 = 8$, $8 \times 7 = 56$*)
- “There are 16 pieces of laundry. There are 8 pieces in each basket.” (*Answers: $16 \div \underline{\hspace{1cm}} = 8$ and $\underline{\hspace{1cm}} \times 8 = 16$, $16 \div 2 = 8$, $2 \times 8 = 16$*)

Prompt your student with the following questions if he is struggling to begin:

- “What information do we know? What information is missing?”
- “Which number in the division equation tells us how many connecting cubes we should start with?”
- “Looking at your model, how many groups are there, and how many are in each group?”

Complete Activity Sheet

“Let’s practice relating division and multiplication.”



Laundry Lineup

Remove pages 223 to 225 from the *Flip into Math!* activity book.

Cut out the images at the bottoms of the pages, and then spread them out in the workspace.

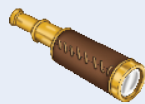
New Teaching

(continued)

“Help Hannah by picking up the laundry. Using the cards, make a multiplication equation that matches the division equation. Use the connecting cubes or make an array to help you find the unknown amount. You will not use all the cards. Once you have created your equation, glue the matching cards to the page.”

Answer Key

- $35 \div 5 = 7$, $5 \times 7 = 35$ or $7 \times 5 = 35$
- $18 \div 2 = 9$, $2 \times 9 = 18$ or $9 \times 2 = 18$
- $24 \div 4 = 6$, $6 \times 4 = 24$ or $4 \times 6 = 24$
- $21 \div 7 = 3$, $3 \times 7 = 21$ or $7 \times 3 = 21$



Look For

When creating the corresponding multiplication equation, your student might write the numbers in the same order. For example, he might make the equation $4 \times \underline{\quad} = 2$ for the division equation $4 \div \underline{\quad} = 2$. He might also be unsure of the unknown amount in the multiplication equation.

Here's How to Help: Help your student understand that the total number being divided represents the product in the related multiplication equation. Have him use the connecting cubes to model each situation and division equation to help him find the multiplication equation and the unknown amount.

Math Reflection

“Let's Reflect!”

Ask some questions to guide your student's reflection:

- “How can you use multiplication to represent a model that shows division?”
- “How are multiplication and division equations similar and different?”
- “What was something you found challenging in this lesson?”

Extended Practice (Optional)

If your student is not able to identify multiplication equations that correspond to diagrams and equations showing division, or he expressed the need for more practice, continue working on this skill.

New Teaching (continued)

Have your student use the connecting cubes to model each of the following division equations or situations. Then, have him write a corresponding multiplication expression and find the unknown value.

- $72 \div \underline{\quad} = 9$ (Answers: $8 \times 9 = 72$, 8)
- $10 \div 2 = \underline{\quad}$ (Answers: $2 \times 5 = 10$, 5)
- Sam placed 11 pieces of laundry into an unknown number of baskets. Each basket has 11 pieces of laundry. (Answers: $1 \times 11 = 11$, 1)

Your student will continue to practice relating multiplication and division in future lessons. You can proceed to the next lesson without full mastery of this skill.

Darby's Math Fun!

Clothes Cleanup

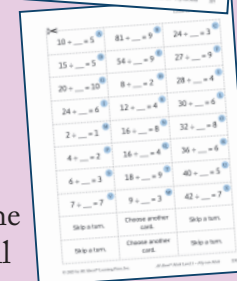
“Let’s practice our math skills!”

Materials

- Clothes Cleanup Gameboards, *Flip into Math!* pages 227 to 229
- Clothes Cleanup Cards, *Flip into Math!* page 231
- Connecting Cubes
- dry-erase board and marker

Directions

1. Each player will get a *Clothes Cleanup* gameboard.
2. Cut apart the *Clothes Cleanup* cards, mix them up, and place them face down in a stack.
3. Player 1 chooses a card and places it face up for all to see. He will use the division equation to make a model using connecting cubes. Then, he will complete the equation and write the related multiplication equation on the dry-erase board.
4. If Player 2 agrees that the multiplication equation is correct, Player 1 turns his card over to reveal the clothing item and puts it into the washing machine on his *Clothes Cleanup* gameboard. If incorrect, he places the card at the bottom of the stack. Player 2 then takes his turn.
5. If a player chooses a card that says, “Choose another card,” he will choose another card and follow steps 3 and 4. If he is correct, both cards are placed into his washing machine. If incorrect, the “Choose another card.” card is set aside, and the other card is put at the bottom of the stack.
6. If a player draws a card that says, “Skip a turn,” the card is set aside, and the player’s turn is over.
7. The first player to place five *Clothes Cleanup* cards into the washing machine is the winner!



Answer Key

A: $2, 10 \div 2 = 5, 2 \times 5 = 10$

B: $9, 81 \div 9 = 9, 9 \times 9 = 81$

C: $8, 24 \div 8 = 3, 8 \times 3 = 24$

D: $3, 15 \div 3 = 5, 3 \times 5 = 15$

E: $6, 54 \div 6 = 9, 9 \times 6 = 54$

F: $3, 27 \div 3 = 9, 3 \times 9 = 27$

G: $2, 20 \div 2 = 10, 2 \times 10 = 20$

H: $4, 8 \div 4 = 2, 4 \times 2 = 8$

I: $7, 28 \div 7 = 4, 7 \times 4 = 28$

J: $4, 24 \div 4 = 6, 4 \times 6 = 24$

K: $3, 12 \div 3 = 4, 3 \times 4 = 12$

L: $5, 30 \div 5 = 6, 5 \times 6 = 30$

M: $2, 2 \div 2 = 1, 2 \times 1 = 2$

N: $2, 16 \div 2 = 8, 2 \times 8 = 16$

O: $4, 32 \div 4 = 8, 4 \times 8 = 32$

P: $2, 4 \div 2 = 2, 2 \times 2 = 4$

Q: $4, 16 \div 4 = 4, 4 \times 4 = 16$

R: $6, 36 \div 6 = 6, 6 \times 6 = 36$

S: $2, 6 \div 2 = 3, 2 \times 3 = 6$

T: $2, 18 \div 2 = 9, 2 \times 9 = 18$

U: $8, 40 \div 8 = 5, 8 \times 5 = 40$

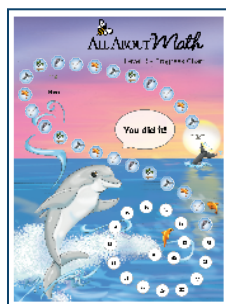
V: $1, 7 \div 1 = 7, 1 \times 7 = 7$

W: $3, 9 \div 3 = 3, 3 \times 3 = 9$

X: $6, 42 \div 6 = 7, 6 \times 7 = 42$

Track Your Progress

Mark the Progress Chart



Have your student mark Lesson 24 on the Progress Chart.

Lesson 31 Multiplying Larger Numbers Within 100

Objective

Your student will learn to multiply a one-digit number by a two-digit number, resulting in a product within 100.

You Will Need

- | | |
|--|--|
| <input type="checkbox"/> <i>Flip into Math!</i> pages 285 to 288 | <input type="checkbox"/> blank sheet of paper |
| <input type="checkbox"/> dry-erase board, pocket, and marker | <input type="checkbox"/> Base-10 Blocks |
| | <input type="checkbox"/> Multiplication Table, <i>Darby's Math Tools</i> |

Before You Begin

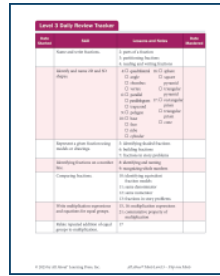
Preview Multiplying One-Digit Numbers by Two-Digit Numbers

In this lesson, your student will practice multiplying a one-digit number by a two-digit number using base-10 blocks and tape diagrams. These methods will help her visualize the multiplication process and break down the numbers into manageable parts.

First, your student will use base-10 blocks to represent the numbers in the equation. Then, she will build groups of tens and ones, allowing her to count the total. This hands-on activity reinforces the idea of place value as she physically manipulates the blocks.

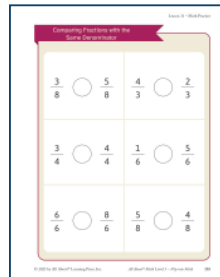
Next, your student will be introduced to a new kind of tape diagram, another visual tool that breaks down multiplication into parts. By drawing a rectangle and dividing it into two sections (one for tens and one for ones), she will begin applying the distributive property. This approach allows her to multiply each part separately by the one-digit number and then combine the partial products to find the final answer.

Throughout the lesson, your student will practice solving problems by using both methods to deepen her understanding of multiplication. This approach will not only strengthen her math skills but also encourage her to think critically and to see math as a visual and logical process.



Daily Review

As part of your math time each day, refer to your student's *Daily Review Tracker*. Choose one or two skills, and take a few minutes to practice.

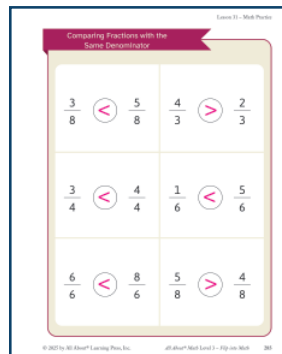


Math Practice

Turn to page 285 in the *Flip into Math!* activity book.

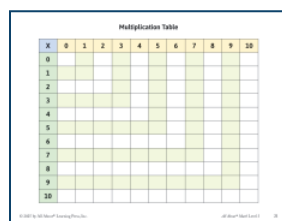
“Let’s review comparing fractions that have the same denominator. Compare each fraction by using a comparison symbol.”

Answer Key



Review Multiplying Multiplication Patterns

“Let’s review what we learned about multiplying by zero and one.”



Multiplication Table

Find the *Multiplication Table* from *Darby's Math Tools*.

“Take a look at your *Multiplication Table*.

What patterns do you notice when multiplying numbers by zero?” *I noticed that the first row and first column in the table are filled with zeros.*

Review (continued)

If your student is unsure where to look on the *Multiplication Table*, guide her to look at the first row and column.

“What do you think that tells us?” *It tells us that multiplying any number by zero always equals zero.*

“Great! Now, let’s think about numbers that are multiplied by one. Take a look at the row and column for one. What do you notice?” *I see that the numbers increase, just like counting.*

“Yes. What do you think that tells us?” *It tells us that any number multiplied by one stays the same.*

New Teaching

Multiplying with Base-10 Blocks

“Today, we’re going to learn about multiplying a one-digit number by a two-digit number.” Write $3 \times 14 = \underline{\quad}$ on the dry-erase board.

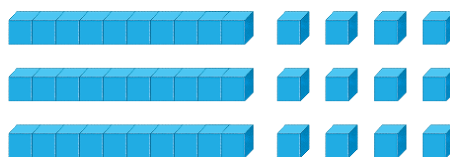
“First, let’s talk about what the numbers in this equation mean. What does the number three represent?” *It means that there are three groups.*

“Great! What does 14 represent?” *The number in each group.*

“Yes. So we need to make 3 rows of 14. Use your base-10 blocks to create the number 14.”



“Good. You created 1 row of 14. Now, make 2 more rows to show 3 rows of 14.”



“Now that we’ve built our groups, let’s count by tens and ones to find the total. First, count the tens.”

Point to each rod as you count along with your student. “10, 20, 30.”

“Good! Now, skip count by 4s from 30.” (Point to each set of 4 units as you count along with your student.) “34, 38, 42.”

New Teaching

(continued)

Write the number 42 on the dry-erase board to complete your equation.
“Yes. The total amount is 42. That means $3 \times 14 = 42$.”

Set this model aside to use again later in the lesson.

“Let’s practice using our base-10 blocks to build a few more problems.”
Have your student build and solve the following expressions by using the base-10 blocks:

- 5×11 (Answer: five rows, each containing one rod and one unit, 55)
- 4×22 (Answer: four rows, each containing two rods and two units, 88)
- 2×32 (Answer: two rows, each containing three rods and two units, 64)

Multiplying with Tape Diagram

“Earlier, you created a model with your base-10 blocks to represent the equation 3×14 . We are going to draw a tape diagram to represent this model.”

Place a sheet of paper into the dry-erase pocket. Then, write the expression 3×14 at the top of the page. “First, I will draw a rectangle and divide it into two parts, one for the tens and one for the ones.”

$$3 \times 14$$



Point to the first factor in the expression. “The first factor in the expression 3×14 is 3. I will write the number 3 to the left of the rectangle.”

$$3 \times 14$$

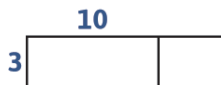


Point to the second factor in the expression.

“Look at the second factor. There is a one in the tens place. What is the value of the one?” *Ten.*

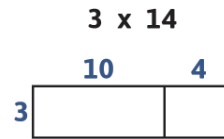
“Yes. We can write the number ten above the first rectangle.”

$$3 \times 14$$

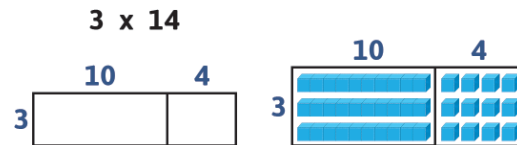


New Teaching (continued)

“There is a four in the ones place. I will write the number four above the second rectangle.”



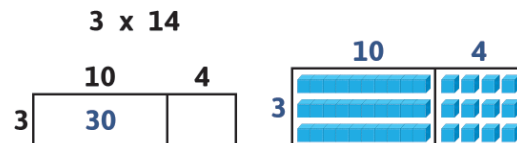
Place the base-10 model onto the dry-erase pocket below to the tape diagram that you drew. Draw rectangles around the rods and the units. Then, write the numbers to match the tape diagram. “Take a look at how our base-10 model is similar to our tape diagram.”



“These rods show three rows of ten. Our tape diagram also shows three rows (point to the three on the tape diagram model) of ten (point to the ten on the tape diagram model). Let’s multiply the factors. What is the value of 3×10 ?” *Thirty.*

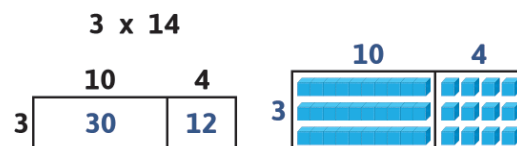
“Good. We can check our answer by counting the rods.” Count the rods by tens with your student.

“We can write the number 30 in the first rectangle.”



“What is the value of 3×4 ?” *Twelve.*

Write the number 12 in the second rectangle. “When we add the numbers 30 (point to the 30) and 12 (point to the 12) together, we can find the value of 3×14 .”



“What is the value of $30 + 12$?” *Forty-two.*

“Yes. That means 3×14 is 42.”

“Now, let’s try another problem.” Write 4×24 on the dry-erase pocket at the top of the page.

New Teaching

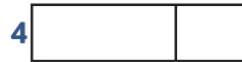
(continued)

“Start by drawing a large rectangle. Since 24 has two place values, divide your rectangle into two parts, one for the tens and one for the ones.”

Point to the first factor. “What is the first factor in our expression?” *Four.*

“Yes. That tells us we will have four equal groups. Write the number four to the left of the rectangle.”

$$4 \times 24$$



Point to the second factor. “Now look at 24. What’s the value of the digit in the tens place?” *Twenty.*

“Good. Write 20 above the first part of the rectangle. What will you write above the next box?” *The next box is for the ones place. I will write four.*

“Write four above the second part of the rectangle.”

$$4 \times 24$$

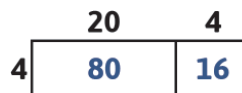


“Now, find the products. What is 4×20 ?” *Eighty.*

“Great job. Write 80 in the first section of the rectangle. What is 4×4 ?” *Sixteen.*

“Yes, write 16 in the second section of the rectangle.”

$$4 \times 24$$



“Now that you have solved both partial products, what is your next step?” *I need to add the partial products together to find the total.*

“Good. What is $80 + 16$?” *Ninty-six.*

“That’s right, so 4×24 equals 96.”

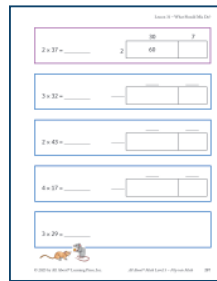
“Let’s practice more problems by using tape diagrams. Draw and use a tape diagram to solve each of the following equations.” If needed, your student can continue using base-10 blocks to make the relationship to the tape diagram more concrete.

New Teaching (continued)

- 2×42 (Answer: 84)
- 3×28 (Answer: 84)
- 4×15 (Answer: 60)
- 5×18 (Answer: 90)

Complete Activity Sheet

“Let’s practice multiplying larger numbers!”



What Should Mia Do?

Remove page 287 from the *Flip into Math!* activity book.

Have your student look at the first problem. “Mia is working on solving the equation $2 \times 37 = \underline{\quad}$. First, she found the value of 2×30 by using a tape diagram.

Explain what Mia should do next to finish finding the value of 2×37 . Then, finish solving the problem. You can use your base-10 blocks to help you finish the tape diagram and solve the problem.”

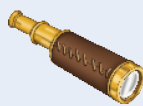
“Next, solve each equation on your page by completing a tape diagram. Then, write your answer on the line next to the equation.”

Answer Key

Mia should find the value of 2×30 and 2×7 , and then add the two products together to solve the equation 2×37 .

(Answer: $2 \times 7 = 14$, $60 + 14 = 74$, $2 \times 37 = 74$)

- $3 \times 32 = 96$
- $2 \times 43 = 86$
- $4 \times 17 = 68$
- $3 \times 29 = 87$



Look For

Your student might overlook the place values of the digits in a two-digit number. For example, when multiplying 2×37 , she might mistakenly treat the 3 in 37 as just 3, leading her to calculate $2 \times 3 = 6$ instead of recognizing that 3 represents 30, making the correct calculation $2 \times 30 = 60$.

Here’s How to Help: Help your student break down the two-digit number into its place value parts. Explain that 37 is made up of 30 and 7, so when multiplying by 2, she needs to multiply each part separately. Guide her through solving this step by step: $2 \times 30 = 60$, $2 \times 7 = 14$, and then add 60 and 14 to get 74. Using visuals, such as base-10 blocks, can help reinforce this concept.

Math Reflection

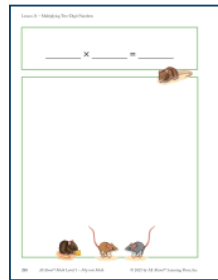
“Let’s Reflect!”

Ask some questions to guide your student’s reflection:

- “How can drawing a tape diagram help you multiply large numbers?”
- “Which strategy is easiest for you to use when solving multiplication problems with two-digit numbers?”
- “What is one thing you would like to continue practicing?”

Extended Practice (Optional)

If your student is struggling to multiply one-digit numbers by two-digit numbers, or if she has expressed the need for more practice, continue reinforcing this skill.



Multiplying Two-Digit Numbers

Flip over the activity page to page 288, and place it into the dry-erase pocket.

For each of the following equations, have your student use base-10 blocks to model the equation, and then have her draw a tape diagram to represent and solve the equation. You can choose to have her complete all the equations or just a few.

- $2 \times 28 =$ (Answer: 56)
- $3 \times 31 =$ (Answer: 93)
- $4 \times 21 =$ (Answer: 84)
- $5 \times 19 =$ (Answer: 95)
- $6 \times 15 =$ (Answer: 90)
- $8 \times 12 =$ (Answer: 96)

Your student will continue to develop her skills with multiplying two-digit numbers in future lessons and daily reviews. You can proceed to the next lesson without full mastery of this skill.

Mark the Daily Review Tracker

Date	Skill	1	2	3	4	5	6	7	8	9	10
	Multiplication										
	Division										
	Addition/Subtraction										

Write today’s date next to the skill: Multiply and divide within 100 using strategies. Include this in your rotation of daily review items.

Three-in-a-Row Multiplication

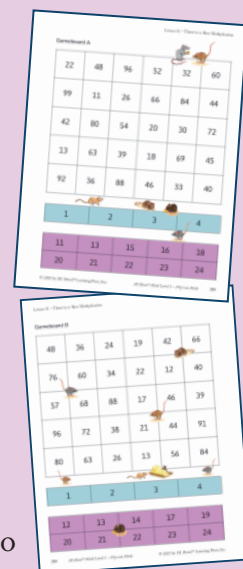
"Let's practice our math skills!"

Materials

- Three-in-a-Row Multiplication Gameboards, *Flip into Math!* pages 289 and 290
- two Unit Cubes
- Two-Color Counters
- blank sheet of paper
- dry-erase pocket and marker

Directions

1. Place a blank sheet of paper into the dry-erase pocket.
2. Choose one of the gameboards to play.
3. Player 1 and Player 2 decide who will use red two-color counters and who will use yellow two-color counters.
4. Player 1 places a unit cube on one of the numbers in the blue section and then places another unit cube on one of the numbers in the purple section. She then multiplies the two factors together.
5. If correct, Player 1 places her two-color counter onto the product on the grid. If incorrect, her turn is over.
6. Player 2 moves one of the unit cubes to another number. Specifically, she moves the unit cube to another number in the blue or the purple section.
7. Player 2 multiplies the two factors to find the product. If correct, she places her two-color counter onto the product on the grid. If incorrect, her turn is over.
8. Players will want to think about their choices carefully and try to choose factors that will equal a product they need. Players also need to know that a few combinations of factors are not used on each gameboard, so choose wisely!
9. Play continues until one player has three two-color counters in a row (horizontally, vertically, or diagonally).



Answer Key

Gameboard A:

$1 \times 11 = 11$	$1 \times 13 = 13$	$1 \times 15 = 15$	$1 \times 16 = 16$	$1 \times 18 = 18$
$1 \times 20 = 20$	$1 \times 21 = 21$	$1 \times 22 = 22$	$1 \times 23 = 23$	$1 \times 24 = 24$
$2 \times 11 = 22$	$2 \times 13 = 26$	$2 \times 15 = 30$	$2 \times 16 = 32$	$2 \times 18 = 36$
$2 \times 20 = 40$	$2 \times 21 = 42$	$2 \times 22 = 44$	$2 \times 23 = 46$	$2 \times 24 = 48$

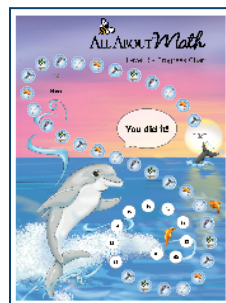
$3 \times 11 = 33$	$3 \times 13 = 39$	$3 \times 15 = 45$	$3 \times 16 = 48$	$3 \times 18 = 54$
$3 \times 20 = 60$	$3 \times 21 = 63$	$3 \times 22 = 66$	$3 \times 23 = 69$	$3 \times 24 = 72$
$4 \times 11 = 44$	$4 \times 13 = 52$	$4 \times 15 = 60$	$4 \times 16 = 64$	$4 \times 18 = 72$
$4 \times 20 = 80$	$4 \times 21 = 84$	$4 \times 22 = 88$	$4 \times 23 = 92$	$4 \times 24 = 96$

Gamboard B:

$1 \times 12 = 12$	$1 \times 13 = 13$	$1 \times 14 = 14$	$1 \times 17 = 17$	$1 \times 19 = 19$
$1 \times 20 = 20$	$1 \times 21 = 21$	$1 \times 22 = 22$	$1 \times 23 = 23$	$1 \times 24 = 24$
$2 \times 12 = 24$	$2 \times 13 = 26$	$2 \times 14 = 28$	$2 \times 17 = 34$	$2 \times 19 = 38$
$2 \times 20 = 40$	$2 \times 21 = 42$	$2 \times 22 = 44$	$2 \times 23 = 46$	$2 \times 24 = 48$
$3 \times 12 = 36$	$3 \times 13 = 39$	$3 \times 14 = 42$	$3 \times 17 = 51$	$3 \times 19 = 57$
$3 \times 20 = 60$	$3 \times 21 = 63$	$3 \times 22 = 66$	$3 \times 23 = 69$	$3 \times 24 = 72$
$4 \times 12 = 48$	$4 \times 13 = 52$	$4 \times 14 = 56$	$4 \times 17 = 68$	$4 \times 19 = 76$
$4 \times 20 = 80$	$4 \times 21 = 84$	$4 \times 22 = 88$	$4 \times 23 = 92$	$4 \times 24 = 96$

Track Your Progress

Mark the Progress Chart



Have your student mark Lesson 31 on the Progress Chart.

Lesson 40 Applying Rounding to Situations

Objective

Your student will learn to decide when rounding is appropriate and then round multi-digit whole numbers within 1,000,000 to solve problems.

You Will Need

- ☐ *Flip into Math!* pages 369 to 372
- ☐ dry-erase board, pocket, and marker
- ☐ Six-Digit Place Value Chart, *Darby's Math Tools*
- ☐ Open Number Line, *Darby's Math Tools* (optional)

Before You Begin

In this lesson, your student will explore how rounding affects problem solving in practical situations. For example, while \$34.43 would typically round down to \$34.00, doing so could result in insufficient money for a purchase. Your student will learn to think critically about the best direction to round a number when given a specific situation.

Your student will learn to understand when rounding is useful and how it can change calculations.

Encourage your student to think about why rounding is helpful in real-life situations. Have him share examples of when people estimate instead of using precise numbers.

Review

Daily Review

As part of your math time each day, refer to your student's *Daily Review Tracker*. Choose one or two skills, and take a few minutes to practice.

Review (continued)



Math Practice

Turn to page 369 in the *Flip into Math!* activity book.

“Let’s review solving story problems. Some story problems will require just one step to solve, while others will need two steps. Also, some problems may include extra information that you do not need to answer the question. Read each story problem carefully, decide what information is important, and solve to find the answer to the question.”

Explain to your student that the \$ symbol indicates dollars. For example, \$25 would be read as “twenty-five dollars.”



Answer Key

- \$44
- 10 communication devices
- 20 oxygen tanks
- 8 cups

Rounding

Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones

Six-Digit Place Value Chart

Find the *Six-Digit Place Value Chart* in *Darby’s Math Tools*, and place it into the dry-erase pocket.

“Write the number 387,450 on the *Six-Digit*

Place Value Chart.”

“What are the two multiples of 1,000 that are closest to the number?” *They are 387,000 and 388,000.*

Your student can use the *Open Number Line* from *Darby’s Math Tools* or draw an open number line on the dry-erase board to help with visualizing how to round larger numbers.



“How would you round this number to the nearest thousand, and what is the rounded number?” *I would look at the digit in the hundreds place. It is a 4, so the digit in the thousands place stays at 7. The number rounds to 387,000.*

Clear the dry-erase pocket.

Rounding in Different Situations

“Imagine you are working in Mission Control during a space mission. Mission Control is a group of people on the ground who manage space flights from before they lift off to when they land back on Earth. They work as a team to monitor and coordinate all the details of a space mission to make sure everything runs smoothly, and they find solutions for any issues that come up during the mission. The team may need to estimate numbers quickly to do this. Let’s help them by rounding numbers in some situations that take place in Mission Control.”

“Mission Control has calculated that a rocket will need 328,746 gallons of fuel for takeoff. We can help by giving an estimate of how much fuel the truck needs to deliver. We will round the number to the nearest thousand.”

“Write the number on the place value chart.”

Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones
3	2	8	7	4	6

“Since we are rounding to the thousand, we can underline the digit in that place. This shows us what place we are rounding to. Underline the digit in the thousands place.”

Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones
3	2	<u>8</u>	7	4	6

“Next, we can circle the number to the right. Circle the number in the hundreds place.”

Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones
3	2	<u>8</u>	7	4	6

New Teaching

(continued)

“Explain how to round this number.” *The place value to the right of the thousands place has a number more than 5. This means it rounds up to a larger number. The number 328,746 rounded to the nearest thousand is 329,000.*

“Why do you think Mission Control might use a rounded number instead of the exact number?” *Rounded numbers are easier to work with.*

“When we round numbers, we simplify them to make them easier to understand and compare. But what if Mission Control rounded the 328,746 gallons of fuel to the nearest hundred thousand instead? Then, they would round it down to 300,000. Is that the best choice?” *Probably not, because the rocket actually needs almost 329,000 gallons of fuel.*

“That’s right! If we round down, we might not have enough fuel for the mission. But what if we round up instead?” *There might be extra fuel.*

“Rounding up to 400,000 gallons ensures that the rocket has enough fuel, which could mean we end up with more than necessary. Rounding is a decision-making tool, not just a math rule. Sometimes, rounding up is the better option, even if the rule says to round down.”

“Mission Control has received a report that the spacecraft will travel 946,387 miles on its journey to take detailed pictures. To help with quick calculations, we need to estimate this number to the nearest ten thousand.”

“Write the number 946,387 on the place value chart.”

Six-Digit Place Value Chart					
Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones
9	4	6	3	8	7

“Underline the number in the place we are rounding to.” Your student should underline the four in the ten thousand place.

“What place will help you know how to round the number?” *The place value to the right, which is the thousands place.*

“Yes. Circle that digit.” Your student should circle the six.

“How do you use the thousands place to help you round this number?” *The number in the thousands place is 6. Six is greater than 5, so I would round up to 950,000.*

New Teaching (continued)

“Do you think rounding up is a good idea for this situation?” *Yes, because Mission Control is looking at distance. If they round down to a smaller number, the crew may run out of fuel or supplies.*

“Let’s try another example. Mission Control is tracking a new space station for monitoring and support. This space station that orbits Earth travels 477,415 miles each day.”

“Write 477,415 on the place value chart.”

Six-Digit Place Value Chart					
Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones
4	7	7	4	1	5

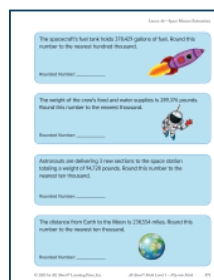
“Round the number to the nearest thousand.” Encourage your student to underline the digit in the place he is rounding to and circle the number he will use to round. Have your student talk through his process, looking at the number to the right and rounding down because the number is less than five.

“If Mission Control decides how much fuel the space station needs on this estimate, what might happen?” *They might not send enough fuel because the space station travels farther than 477,000 miles per day.*

“Rounding is not just about finding the nearest ten, hundred, or thousand. It is about making decisions based on what the number represents.”

Complete Activity Sheet

“Let’s practice rounding.”



The activity sheet titled "Space Mission Estimations" contains four rounding exercises, each with a small illustration and a "Rounded Number" line.

- Exercise 1: "The spaceport's fuel tank holds 215,425 gallons of fuel. Round this number to the nearest hundred thousand." (Illustration of a rocket launch)
- Exercise 2: "The weight of the crew's food and water supplies is 285,274 pounds. Round this number to the nearest thousand." (Illustration of astronauts in a space station)
- Exercise 3: "Astronauts are collecting 1,100 samples in the space station. Round this number to the nearest hundred." (Illustration of a space station orbiting Earth)
- Exercise 4: "The distance from Earth to the Moon is 238,855 miles. Round this number to the nearest ten thousand." (Illustration of Earth from space)

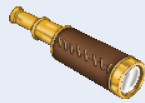
Space Mission Estimations

Turn to page 371 in the *Flip into Math!* activity book.

“A team of astronauts is preparing for a deep-space mission, and they need accurate estimations to ensure a safe journey. Mission Control has collected key data about the spacecraft’s fuel, distance to checkpoints, and the weight of supplies. The astronauts need easy-to-use estimates. You can help by rounding each number to the correct place value. Then, explain whether the rounded number would be good to use as an estimate.”

Answer Key

400,000; 289,000; 90,000; 240,000



Look For

Some students may only focus on the digit he is rounding rather than checking the place value one digit to the right. This can lead to incorrect rounding, such as rounding 378,429 to 300,000 instead of 400,000.

Here's How to Help: Encourage your student to write the number in a place value chart and underline the digit in the place he is rounding to. Remind him to look at the digit immediately to the right before deciding whether to round up or down. If he is unsure, ask him to identify the 2 closest multiples (such as 300,000 and 400,000) and decide which one the number is closer to.

Math Reflection

“Let’s Reflect!”

Ask some questions to guide your student’s reflection:

- “When would you round numbers instead of using exact values?”
- “Can rounding ever lead to a mistake?”
- “When might exact numbers be better?”

Extended Practice (Optional)

If your student is not able to round numbers to the nearest thousand, ten thousand, or hundred thousand, or he expressed the need for more practice, continue working on this skill.

Number	Round to the Nearest Thousand	Round to the Nearest Ten Thousand	Round to the Nearest Hundred Thousand
462,734	463,000	460,000	400,000
128,714	129,000	130,000	100,000
874,621	875,000	870,000	900,000
285,188	285,000	290,000	300,000
971,831	972,000	970,000	1,000,000
127,884	128,000	130,000	100,000
412,897	413,000	410,000	400,000
794,248	794,000	790,000	800,000


Rounding Match-Up

Turn to page 372 in the *Flip into Math!* activity book.

“A space research team is analyzing large data sets for their latest mission. However, some of the recorded values need to be rounded to make calculations easier! Help the team match each original number with its correctly rounded value.”

Lesson 40 – Rounding Match-Up

Number	Place Value to round to:	Choose the number that is rounded correctly.		
462,739	ten thousand	460,000	470,000	500,000
528,194	hundred thousand	500,000	530,000	600,000
874,623	ten thousand	870,000	875,000	880,000
249,358	thousand	250,000	249,000	240,000
391,826	hundred thousand	300,000	390,000	400,000
527,894	ten thousand	500,000	520,000	530,000
413,870	ten thousand	400,000	410,000	414,000
794,308	hundred thousand	700,000	790,000	800,000



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Darby's Math Fun!

Rounding Challenge

“Let’s practice our math skills!”

Materials

- Rounding Challenge Cards, *Flip into Math!* page 373

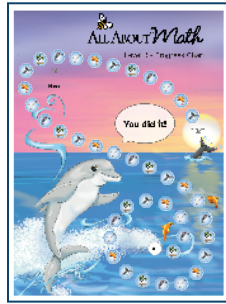
Directions

1. Cut apart the *Rounding Challenge* cards.
2. Find the “Start here” card, and place it to the side.
3. Mix up the remaining cards, and spread them out with the number side facing up.
4. A player will read the starting question.
5. Both players will look for the card showing the answer. The first player to find the card that rounds to the number picks up the card and reads the number. The other player will agree on whether it is correct or incorrect.
6. If correct, that player will read the question on the back of the number card he just found and will keep the card. If incorrect, the player returns the card to the workspace.
7. Play continues as players select the correct number and read the next question.
8. When all cards have been used, each player will count his cards. The player with the most cards is the winner!



Track Your Progress

Mark the Progress Chart



Have your student mark Lesson 40 on the Progress Chart.

This is it - our last
lesson together is coming up!
It has been so much fun sharing your
world with you and
watching you learn so much.

I am so proud of all that you
have accomplished and
you should be, too!



APPENDIX A

Scope and Sequence of Level 3

Your Student Will:	Lesson
Review skills and concepts involving adding and subtracting within 100 and reading and writing six-digit numbers.	1
Learn to identify fractions from wholes that have already been partitioned into equal parts.	2
Learn to draw and partition shapes into equal parts.	3
Learn to read and write fractions that represent parts of a whole.	4
Learn to identify shaded fraction models for a given fraction.	5
Learn to create a model to represent a fraction.	6
Learn to represent fractional situations by using diagrams.	7
Learn to identify and name fractions on a number line.	8
Learn to recognize fractions that are equivalent to whole numbers and to write whole numbers as fractions.	9
Learn to identify equivalent fractions.	10
Learn to compare two fractions that have the same denominator.	11
Learn to compare two fractions that have the same numerator.	12
Learn to learn to compare two fractions with the same numerator or denominator in story problems and explain the conclusions.	13
Review all concepts learned in Lessons 2–13.	14
Identify and draw equal groups to learn about the concept of multiplication.	15
Learn to use multiplication expressions to represent equal groups.	16
Learn about the relationship between repeated addition and multiplication, and write expressions to represent each.	17

Your Student Will:	Lesson
Learn to represent multiplication situations with arrays.	18
Learn to connect multiplication arrays to multiplication expressions and equations.	19
Learn to represent and solve multiplication story problems by writing equations with unknown factors or products, and by creating arrays.	20
Be introduced to the commutative property of multiplication.	21
Review all concepts learned in Lessons 2–21.	22
Learn to relate division to multiplication and to recognize division as an unknown-factor problem.	23
Learn to relate multiplication and division and to recognize division as an unknown-factor problem.	24
Learn to use the relationship between multiplication and division to write equations.	25
Learn to identify multiplication and division equations that represent arrays.	26
Learn to represent and solve story problems that involve equal groups by using multiplication and division equations.	27
Learn to identify single-digit multiplication facts and the related division facts.	28
Review all concepts learned in Lessons 2–28.	29
Learn to identify and explain patterns in the multiplication table.	30
Learn to multiply a one-digit number by a two-digit number, resulting in a product within 100.	31
Learn to use strategies to divide within 100.	32
Learn to analyze two-step problems to identify necessary and unnecessary information.	33
Learn to represent and solve two-step story problems using the four operations.	34
Review all concepts learned in Lessons 2–34.	35

Your Student Will:	Lesson
Learn to read and write nine-digit whole numbers in standard form and identify the place and value of each digit.	36
Learn to compare 2 multi-digit whole numbers within 1,000,000 using place value reasoning.	37
Learn to order multi-digit whole numbers within 1,000,000.	38
Learn to round multi-digit whole numbers within 1,000,000 to the nearest multiple of thousand, ten thousand, and one hundred thousand.	39
Learn to decide when rounding is appropriate and then round multi-digit whole numbers within 1,000,000 to solve problems.	40
Review all concepts learned in Lessons 2–40.	41

APPENDIX C

Guidelines for Using Manipulatives and Drawings

Manipulatives and drawings are essential tools for building a strong foundational understanding of math concepts, such as addition, subtraction, multiplication, and division. These tools help children develop abstract thinking skills and concretely understand what operations mean (e.g., counting objects to understand addition). It is important to allow students to use manipulatives and drawings as long as they find them helpful or beneficial.

- **Children learn best by engaging with learning through multiple senses**, using sight, sound, and touch. Kinesthetic learning, meaning learning by touch, allows students to physically interact with the materials. Manipulatives provide a tangible way for learners to interact with mathematical concepts, allowing them to internalize ideas through hands-on exploration. Visual learning is engaged through drawings and diagrams, providing an essential understanding of relationships between numbers, geometric shapes, and patterns. Being able to draw helps learners organize information more effectively.
- **Manipulatives and drawings are powerful tools for reinforcing concepts over time.** By continuing to use them, children can deepen their understanding and retain knowledge better than if they were forced to rely solely on abstract methods.
- **When children continue to use physical tools to explore math, they develop a deeper,** more lasting understanding of why math works the way it does, instead of just memorizing procedures. This understanding is crucial for applying math in real-world situations.
- **Manipulatives and drawings can help make the learning process more enjoyable**, engaging, and hands-on. By encouraging students to use these tools as long as they find them helpful, educators can keep math fun and interesting, which is crucial for fostering a positive attitude toward the subject.
- **Manipulatives and drawings provide scaffolding for students who need more support.** They allow students to progress at their own pace, providing them with a way to fully engage with the material. The goal is to ensure that children are equipped with the support they need at every level of their learning and not to limit or rush their use of these tools.

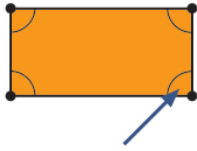
As students become more confident in their ability to perform basic calculations, they can begin to rely less on manipulatives and drawings and start to use more mental math and number-based strategies. However, the exact timing for this transition can vary depending on the child's age and developmental stage, the complexity of the mathematical ideas involved, and the student's level of comfort. In general, students should stop using manipulatives and drawings for basic operations when they have developed fluency and can understand the underlying concepts abstractly. This typically occurs around late elementary or early middle school, but the exact timing can vary. (And for more advanced mathematics, research has shown that manipulatives improve learning for teens and even adults in subjects such as algebra and calculus.) The goal is for them to develop the ability

to think and reason abstractly, while still recognizing that manipulatives and drawings can be useful in more complex or unfamiliar contexts. It is important that students make the decision about when to stop using manipulatives and drawings based on their individual level of comfort with the mathematical concepts involved.

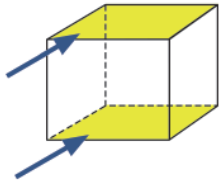
If students do not want to use manipulatives or drawings when the lesson instruction directs them to use these methods, have them demonstrate their understanding of the method by solving 2–3 examples. Then, allow students to solve the remaining examples using the method of their choice. Students will use many of the same methods in later levels to represent increasingly difficult concepts, so it is important that students understand and can apply different manipulatives and drawings to solve problems. However, this must be balanced with preventing students from becoming frustrated by having to use manipulatives and drawings when they can fluently solve problems using mental math and other strategies.

APPENDIX E

2D and 3D Shapes Glossary



Angle: An angle is made when two lines meet at a point.



Base: A base is the bottom or flat part of a shape that it sits on.



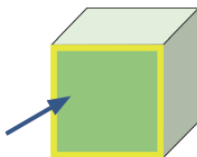
Cone: A cone is a solid shape that has a round base and a point at the top.



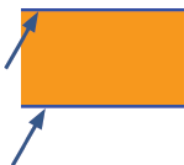
Cube: A cube is a solid shape with six square sides that are all the same size.



Cylinder: A cylinder is a solid shape with two round, flat ends and one curved side.



Face: A face is a flat surface on a solid shape.



Parallel: Parallel lines are lines that are always the same distance apart and never touch.



Parallelogram: A parallelogram is a shape with four sides where opposite sides are the same length and run parallel to each other.



Polygon: A polygon is a flat shape with three or more straight sides that are all connected.



Quadrilateral: A quadrilateral is a shape with four straight sides.



Rectangular prism: A rectangular prism is a solid shape with 6 flat sides that are all rectangles.



Rhombus: A rhombus is a shape with four equal sides and opposite sides that are parallel.



Sphere: A sphere is a solid shape that is round all over.



Square pyramid: A square pyramid is a solid shape with a square base and four triangle sides that meet at a point on top.



Trapezoid: A trapezoid is a shape with four sides, and only one pair of sides are parallel.



Triangular prism: A triangular prism is a solid shape with two triangle ends and three rectangle sides.



Triangular pyramid: A triangular pyramid is a solid shape with a triangle base and three triangle sides that come together at a point on top.



Vertex: A vertex is a corner where two or more lines or edges meet on a shape.